Superfield $S(x, \theta, \theta^\dagger)$

$$S \to S(x^\mu + i\varepsilon\sigma^\mu\theta^\dagger + i\varepsilon^\dagger\bar{\sigma}^\mu\theta, \theta+\varepsilon, \theta^\dagger+\varepsilon^\dagger)$$

$$\hat{Q}_\alpha = i\frac{\partial}{\partial\theta^\alpha} - (\sigma^\mu\theta^\dagger)_\alpha \partial_\mu$$

$$\hat{Q}^{\dagger\dot{\alpha}} = i\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - (\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu$$

$$\sqrt{2}\,\delta_\varepsilon S = -i(\varepsilon\hat{Q} + \varepsilon^\dagger\hat{Q}^\dagger)S$$

(remember the lowering & raising with $\varepsilon^{\alpha\beta}$ $\varepsilon_{\dot{\alpha}\dot{\beta}}$ ...)

But $S = a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c + \theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger$
$+ \theta\theta\theta^\dagger\theta^\dagger d$

(remember the conventions for contractions)

S has too many fields:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu$$

: Chiral (super) covariant derivatives

$$D^{\dagger\dot{\alpha}} = \frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} - i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu$$

$$\{\hat{Q}, D\} = \{\hat{Q}^\dagger, D\} = \{\hat{Q}, D^\dagger\} = \{\hat{Q}^\dagger, D^\dagger\} = 0$$

$$\{D_\alpha, D^\dagger_{\dot{\beta}}\} = 2i\,\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu$$

$$\{D_\alpha, D_\beta\} = \{D^{\dagger\dot{\alpha}}, D^{\dagger\dot{\beta}}\} = 0$$

$$\Rightarrow \quad D_\alpha S' = 0 \quad \Rightarrow \quad D_\alpha(\delta_\varepsilon S') = 0$$

or this allows for a constraint on superfields that removes components

$$\boxed{D^{\dagger\dot\alpha}\Phi = 0 \qquad \text{or} \qquad D_\alpha \Phi^* = 0}$$

↳ chiral (left) antichiral (right) superfield

Choose as coordinates instead $y^\mu = x^\mu + i\theta^\dagger\bar\sigma^\mu\theta$, $\theta, \theta^\dagger$

$y^{*\mu} = x^\mu - i\theta^\dagger\bar\sigma^\mu\theta$

then
$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\mu\theta^\dagger)_\alpha\frac{\partial}{\partial y^\mu} = \frac{\partial}{\partial\theta^\alpha}$$

$$D^{\dagger\dot\alpha} = \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} = \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}} - 2i(\bar\sigma^\mu\theta)^{\dot\alpha}\frac{\partial}{\partial y^{*\mu}}$$

$y^\mu, \theta, \theta^\dagger$ $\qquad\qquad$ y^*, θ, θ

$$\Phi = \phi(y) + \sqrt2\,\theta\,\psi(y) + \theta\theta\,F(y)$$

$$= \phi(x) + i\partial_\mu\phi(x)\,\theta^\dagger\bar\sigma^\mu\theta + \frac{-1}{2}\partial_\mu\partial_\nu\phi(x)\,\theta^\dagger\bar\sigma^\mu\theta\,\theta^\dagger\bar\sigma^\nu\theta$$

$$+ \sqrt2\,\theta\,\psi(x) + \sqrt2\,\theta\, i(\theta^\dagger\bar\sigma^\mu\theta)\partial_\mu\psi(x) + \theta\theta\,F(x)$$

$a = \phi$ $\qquad$ $v_\mu = i\partial_\mu\phi$

$\xi = \sqrt2\,\psi$ $\qquad$ $\eta = 0$

$\chi^\dagger = 0$

$b = F$ $\qquad$ $\zeta^{\dagger\dot\alpha} = \frac{-i}{\sqrt2}(\bar\sigma^\mu\partial_\mu\psi)^{\dot\alpha}$

$c = 0$

$d = \frac{1}{4}\partial_\mu\partial^\mu\phi$

$\Phi = (\bar{D}^\dagger \bar{D}^\dagger) S'$ is always chiral

Another constraint is

$$V = V^* \quad \text{then} \quad \begin{aligned} a &= a^* && \text{real} \\ \chi^\dagger &= \xi^\dagger && \\ c &= b^* && \text{complex} \\ v_\mu &= v_\mu^* && \text{real vector} \\ \zeta^\dagger &= \eta^\dagger && \\ d &= d^* && \text{real} \end{aligned}$$

vector multiplet was λ, v^μ, D

so need to remove a_{real}, ξ, b

set Ω a chiral superfield + require independence under

$$V \to V + i(\Omega^* - \Omega) \qquad (\text{compatible with } V = V^*)$$

this precisely allows to remove a_{real}, ξ, b from V

and $A_\mu \to v_\mu + \partial_\mu(\phi + \phi^*)$ ← usual gauge transformation

+ by convention
$$\begin{cases} \eta_\alpha = \lambda_\alpha - \frac{i}{2}(\sigma^\mu \partial_\mu \xi)_\alpha \\ v_\mu = A_\mu \\ d = \frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a \end{cases}$$
gives our $\lambda, A_\mu + D$

Lagrangians

$$\delta_\varepsilon A = 0 \quad \text{for} \quad A = \int d^4x\, d^2\theta\, d^2\theta^+\, S$$

- D term is $[V]_D = \mathcal{L} = \int d^2\theta\, d^2\theta^+\, V(x,\theta,\theta^+) = \frac{1}{2} D + \frac{1}{4} \partial_\mu \partial^\mu a$
- F term is $\mathcal{L} = \int d^2\theta\, \Phi\Big|_{\theta^+=0} = \int d^2\theta\, d^2\theta^+\, \theta^+\theta^+ \Phi \quad + h.c.!$

$$= F + F^*$$

- Note that $[V]_D = \frac{-1}{4} [D^+ D^+ V]_F$

  using p.i. & $D^+ D^+ (\theta^+\theta^+) = -4$

- $[\bar{\Phi}^{xi}\, \Phi_j]_D$ is the kinetic term

  $[W[\Phi_i]]_F$ gives the term from the superpotentials

- Gauge theory

  Define $W_\alpha = \frac{-1}{4} (D^+ D^+) D_\alpha V \qquad W^{+\dot\alpha} = \frac{-1}{4} (DD) D^{+\dot\alpha} V$

  $\to W_\alpha$ under gauge transformations

gauge multiplet: $\int d^4x\, \mathcal{L} = \int d^4x \left[\frac{1}{4} (W^\alpha W_\alpha) + \frac{1}{4} (W^+_{\dot\alpha} W^{+\dot\alpha}) \right]_F$

& $\quad -k[V]_D$ is also a good term (Fayet - Iliopoulos term)

$$\mathcal{L}(x) = \int d^2\theta\, d^2\theta^+ \left[\frac{1}{4}\left(W^{\alpha} D_\alpha V + W^{+}_{\dot\alpha} D^{+\dot\alpha} V \right) - 2 K V \right]$$

is it as a D term

Gauge invariance + chiral superfields

$$V \to V + i\left(\Omega^* - \Omega\right) \qquad (\Omega = \text{chiral})$$

$$\Phi \to e^{2ig\overset{\to \text{charge}}{q}\,\Omega}\, \Phi \qquad \text{remains chiral !}$$

$\left[\Phi^* e^{2gqV} \Phi \right]$ is invariant under the (super) gauge transformation

Nonabelian: promote everything like matrices & vectors in group space

$$V = 2g\, T^a V^a$$

$$e^V \to e^{i\Omega^+} e^V e^{i\Omega}$$

$$W_\alpha = \frac{-1}{4} D^+ D^+ \left(e^{-V} D_\alpha e^V \right)$$

$$\mathcal{L} = \frac{1}{16\pi i} \tau \operatorname{tr}\left(\hat{W} \hat{W} \right)_F + \text{c.c.} + \left(\Phi^* e^{2g\hat{V}} \Phi \right)$$

the $\wedge$ have an extra g $\qquad \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

+ of course the generalization of more gauge groups

R symmetries

$$\theta \to e^{i\alpha}\theta \qquad \theta^\dagger \to e^{-i\alpha}\theta^\dagger$$

$$\hat{Q} \to e^{-i\alpha}\hat{Q} \qquad [R, Q] = -Q \qquad [R, Q^\dagger] = Q^\dagger$$

So the different components have different charges.

if $\Phi \to e^{i r_\Phi \alpha}\Phi$ $\qquad$ $\phi \to e^{i(r_\phi)\alpha}\phi$

$\psi \qquad (r_\phi - 1)$

$F \qquad (r_\phi - 2)$

V : only $\lambda \to e^{i\alpha}\lambda$ $\qquad$ $A_\mu \to A_\mu$ since they are real

$D \to D$

Nonrenormalizable : more terms are possible