Soft (dim < 4) susy breaking interactions

Vector + chiral multiplets

Vector: put the breaking stuff in the sfermion

Chiral: put the breaking as much as possible in the scalars
(rest can be done by adjusting the superfield interactions)

$$
\begin{aligned}
\mathcal{L}_{soft} = & -\frac{1}{2} M^g \, tr(\lambda^g \lambda^g) \\
& - t^i \phi_i \\
& - \frac{1}{2} b^{ij} \phi_i \phi_j \\
& - \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k \\
& \qquad + h.c. \\
& - (m^2)^i{}_j \, \phi^{*j} \phi_i \\
& - \frac{1}{2} c_i{}^{jk} \phi^{*i} \phi_j \phi_k + h.c. \\
& - M^a \lambda^a \psi_a + h.c.
\end{aligned}
$$

{ or different Majorana mass for each gaugino

} can spoil genetic divergence cancellations if fermions in adjoint group representations present

Last 2 not relevant for MSSM : no gauge invariant ones can be formed

can be relevant for extensions

MSSM

B, W, G vector superfield

$\bar{e}, \bar{u}, \bar{d}$, Q, L, H_u, H_d superfields

need 2 Higgs doublets: • no susy invariant way to give mass to both quarks & leptons upper & lower elements of doublets

$$H_u = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad H_d = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$ both needed

• but also needed to cancel anomalies!

Superpotential

$$\bar{u}\, y_{up}\, Q H_u - \bar{d}\, y_{down}\, Q H_d - \bar{e}\, y_{lepton}\, L H_d + \mu H_u H_d$$

note $\begin{cases} \bullet\ \varepsilon^{\dot{\alpha}\dot{\beta}} \text{ for the } SU(2)_L \text{ index contraction is assumed} \\ \bullet\ \mu \text{ can be made real by changing a phase of } H_u \text{ or } H_d \\ \bullet\ y_{up}, y_{down}, y_{lepton} \text{: redefining as in first lecture possible} \end{cases}$

Note • the μ term a susy invariant mass scale!

$\to$ why small $\ll M_p$? the μ-problem

• Yukawa couplings : see our first lecture

(small except y_t, y_b, y_τ the 3 3 elements of y_{up}, y_{down} and y_{lepton})

$\boxed{\text{Now}}$ W = superpotential

$$W^i_j = \frac{\partial W}{\partial \phi_i}$$

$$V(\phi, \phi^*) = W^*_i W^i + \frac{1}{2} \sum_g g^2_g \sum_a (\phi^* T^a \phi)^2$$

So $V(\phi,\phi^*) \geq 0$

and $W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$

W must be a gauge singlet MSSM : $L^i = 0$

$\Rightarrow$ $V(\phi = \phi^* = 0) = 0$ are
- no Susy breaking
- but also no $SU(2)_L \times U(1)_Y$ breaking

$\boxed{\text{Also}}$

susyterms violating B & L

L: $\frac{1}{2} \lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \underbrace{\mu'^i L_i H_u}$

note:
- L, H_u have the same quantum numbers
- using L for H_u doesn't work either: way too large lepton number violation

B $\frac{1}{2} \lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$

lead to $\underbrace{\mu \to e \gamma}_{> 10^6 \text{ sec}}$ & $\underbrace{p \to e^+ \pi^0}_{> 10^{33} \text{ years}}$ etc of μs lifetimes

- remove by <u>matterparity</u> $P_M = (-1)^{3(B-L)}$: $\bar{u}, \bar{d}, e, Q, L$: -1
  H_u, H_d, B, W, G : $+1$

- always a good symmetry is $(-1)^{2s}$
  (a fermion remains a fermion)

combining the two gives R parity $P_R = (-1)^{3(B-L)+2s}$
where SM : $+1$
susy extra : -1

Most of susy phenomenology assumes R or M parity

- If R parity :
  - the lightest susy particle is stable (LSP)
    (can annihilate but not decay)
  - susy particles produced in pairs
  - susy decays always contain an odd number of LSP

- If LSP has strong or electromagnetic interactions
  => easily detectable (but not fully ruled out I think)
  if not => can be dark matter candidate

Soft Susy breaking terms :

$$-\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B}\right) + h.c.$$

$$-\left(\tilde{\bar{u}}\, a_{up}\tilde{Q}H_u - \tilde{\bar{d}}\, a_{down}\tilde{Q}H_d - \tilde{\bar{e}}\, a_{lepton}\tilde{L}H_d\right) + h.c.$$

$$-\tilde{Q}^{+} m^2_Q \tilde{Q} - \tilde{L}^{+} m^2_L \tilde{L} - \tilde{\bar{u}}^{+} m^2_{\bar{u}} \bar{u} - \tilde{\bar{d}}^{+} m^2_{\bar{d}} d - \tilde{\bar{e}}^{+} m^2_{\bar{e}} \tilde{\bar{e}}$$

$$-m^2_{H_u} H_u^{*} H_u - m^2_{H_d} H_d^{*} H_d - b\left(H_u H_d + b^{*} H_u^{*} H_d^{*}\right)$$

- all other soft terms violate $SU(3)\times SU(2)\times U(1)$ eg $\tilde{L}\tilde{L}$ has a nonzero y

3 real M_i
3 complex 3×3 a
5 hermitian 3×3 m^2

- neglects the c_i^{jk} type of couplings, otherwise matrices with $H_u \to H_d^{*}$ & $H_d \to H_u^{*}$ would be allowed as well giving 3 more complex "a" matrices

- in principle has 105 additional free parameters
  + 54 more if the c_i^{jk} are allowed
- when adding neutrino masses/ right handed neutrinos there are even more terms allowed

- $\mu \to e\gamma$ limits (and also from $K \to \mu e$ ...)
  $\tau \to \mu\gamma$
  $\tau \to e\gamma$
- $K^0 - \overline{K^0}$, $B^0 - \overline{B^0}$, $D^0 - \overline{D^0}$, $B^0_s - \overline{B^0_s}$
- all the flavour changing decays

$\Rightarrow$ all flavour changing parts are strongly suppressed

usual assumptions: (1) $m^2_Q = m^2_Q \mathbb{1}$, ...

so one universal mass for each sparticle

(2) $a_{up} = A_{up}\, y_{up}$, ...

the Yukawas line up

(3) $M_1, M_2, M_3, A_{up}, A_{down}, A_{lepton}$ are real

(Other assumptions possible but then the sparticles need to be heavier but games can be played with the limits)

Then: $\begin{cases} \cdot\ 5 \text{ sparticle masses} \\ \cdot\ 3 \text{ gaugino masses} \\ \cdot\ 3 \text{ "Yukawa proportionality factors} \\ \cdot\ m^2_{H_u},\ m^2_{H_d},\ b \end{cases}$

Renormalization group running

- all of the couplings run
- the patterns assumed are not fully invariant under ~~gauge~~ running
  - so if the pattern is assumed at some scale: check how well it holds at the weak scale.
- Unification of couplings typically a little above 10^{16} GeV

Popular susy breaking models

Planck scale or gravity mediated

PMSB or GMSB

- $M_1 = M_2 = M_3 = m_{1/2}$
- $m_Q^2 = m_u^2 = m_d^2 = m_e^2 = m_L^2 = m_0^2 \, \mathbb{1}$ $\qquad m_{H_u}^2 = m_{H_d}^2 = m_0^2$
- $a_{up} = A_0 \, y_{up}$ same for all
- $b = B_0 \mu$?

at some large scale: $Q \approx M_P$ or $M_{unification}$

so extra susy parameters are $m_{1/2}, m_0^2, A_0, B_0, \mu$

called minimal Sugra : MSUGRA
constrained MSSM : CMSSM

(gravitino) $m_{3/2}$

models with stronger assumptions exist as well.

- dilaton $\quad m_0^2 = m_{3/2}^2 \qquad m_{1/2} = -A_0 = \sqrt{3}\, m_0$
- No scale $\quad m_{1/2} \gg m_0, A_0, m_{3/2}$

$\boxed{\text{Gauge mediated}}$ GMSB

- add extra $q, \bar{q}$ $\quad$ $l, \bar{l}$

$$(3, 1, -\tfrac{2}{3}) \qquad (1, 2, \tfrac{1}{2})$$
$$+\ \bar{q} \qquad\qquad +\ \bar{l}$$
$$(\bar{3}, 1, \tfrac{2}{3}) \qquad (1, 2, -\tfrac{1}{2})$$

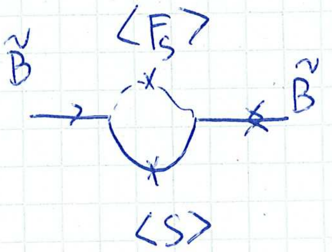
Dirac mass terms are allowed: heavy!

+ gauge singlet S which breaks susy via a vev

$$W_{mess} = y_2\, S \bar{l} l + y_3\, S q \bar{q}$$

S a vev for $\langle \phi_S \rangle$ & $\langle F_S \rangle$

gauginos:



: gaugino masses:

$$M_a = \frac{\alpha_a}{4\pi} \Lambda \qquad \text{masses} \propto g_a^2$$

sparticles: masses at 2-loop

$$m^2_{\phi_i} = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3(i) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2(i) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1(i) \right]$$

Casimirs

& $a_{up} = a_{down} = a_{lepton} \simeq 0$

- by adding more messengers different patterns can be produced.
  + different masses for them

LSP = gravitino

- extra dimensional mediated susy breaking   XMSB
- anomaly mediated " "   AMSB
  (but this needs extra pieces to work)

.... and you can combine all of them

Phenomenology

Higgs potential

$$(|\mu|^2 + m^2_{H_u})(|H_u^0|^2 + |H_u^+|^2)$$
$$(|\mu|^2 + m^2_{H_d})(|H_d^0|^2 + |H_d^-|^2)$$

$$+ \frac{1}{8}(g^2 + g'^2)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2$$
$$+ \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2$$
$$+ b\,(H_u^+ H_d^- - H_u^0 H_d^0) + b^* (H_u^- H_d^+ - H_u^{0*} H_d^*)$$

use gauge invariance: minimum is at $H_u^+ = 0$

potential is fully symmetric under $H_u^+ \leftrightarrow H_d^-$ ~~H^0~~

so $H_d^+ = 0$ is also a minimum

alternatively check $\frac{\partial V}{\partial H_d^+} = 0$ at $H_d^+ = 0 \Rightarrow \frac{\partial V}{\partial H_d^-} = 0$ at $H_d^- = 0$

$$V = (|\mu|^2 + m^2_{H_u})|H_u^0|^2 + (|\mu|^2 + m^2_{H_u})|H_d^0|^2$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$
$$- b H_u^0 H_d^0 - b^* H_u^{0*} H_d^{0*}$$

redefine phase of $H_u^0 \Rightarrow b$ real

V must be bounded from below

$$H_u^0 = H_d^0 \Rightarrow \boxed{2|\mu|^2 + m^2_{H_u} + m^2_{H_d} - 2b > 0}$$

$H_u^0 = H_d^0 = 0$ must not be a stable minimum

both vevs + b can be chosen real + positive at the minimum
(the minimum must be at $b H_u^0 H_d^0$ real.)

$$\begin{pmatrix} |\mu|^2 + m^2_{H_u} & -b \\ -b & |\mu|^2 + m^2_{H_u} \end{pmatrix}$$

one eigenvalue < 0
(sum is positive!)

$$\Rightarrow \boxed{(|\mu|^2 + m^2_{H_u})(|\mu|^2 + m^2_{H_d}) - b^2 < 0}$$

$$v_u^2 + v_d^2 = v^2 = \frac{2 m_Z^2}{g^2 + g'^2} = (174\ \text{GeV})^2$$ (beware $\sqrt{2}$'s)

$$\tan\beta = \frac{v_u}{v_d}$$

$$\frac{\partial V}{\partial H_u^0} = 0 \qquad 2 v_u (|\mu|^2 + m^2_{H_u}) + \frac{1}{8}(g^2 + g'^2)\, 2 v_u (v_u^2 - v_d^2) - 2 b v_d = 0$$

$$\frac{\partial V}{\partial H_d^0} = 0 \qquad 2 v_d (|\mu|^2 + m^2_{H_u}) - \frac{1}{8}\, g^2 + g'^2\, 2 v_d (v_u^2 - v_d^2) - 2 b v_u = 0$$

$$\frac{\partial^2 V}{\partial H_u^{0\,2}} = 2\left(|\mu|^2 + m^2_{H_u}\right) + \frac{(g^2+g'^2)}{8}\left[2\left(v_u^2 - v_d^2\right) + 4v_u^2\right]$$

$$\frac{\partial^2 V}{\partial H_d^{0\,2}} = 2\left(|\mu|^2 + m^2_{H_d}\right) + \frac{(g^2+g'^2)}{8}\left[2\left(v_d^2 - v_u^2\right) + 4v_d^2\right]$$

$$\frac{\partial^2 V}{\partial H_u^0 H_d^0} = -\frac{1}{8}\left(g^2+g'^2\right) 4 v_u v_d - 2b$$

then
- $H^\pm$ mix from H_u^+, H_d^- with $\sin\beta$
- A^0 $\quad \mathrm{Im}\, H_u^0$, $\mathrm{Im}\, H_d^0$ $\quad \sin\beta$

} the other is the eaten Goldstone Boson

and
$$\begin{cases} m^2_{A^0} = 2|\mu|^2 + m^2_{H_u} + m^2_{H_d} \\ m^2_{h^0, H^0} = \frac{1}{2}\left(m^2_{A^0} + m^2_Z \mp \sqrt{\left(m^2_{A^0} - m^2_Z\right)^2 + 4 m_Z^2 m_A^2 \sin^2(2\beta)}\right) \\ m^2_{H^\pm} = m^2_{A^0} + m^2_W \end{cases}$$

$$\Rightarrow \boxed{m^2_{h^0} < m^2_Z}$$

α = h^0, H^0 mixing angle

$\Rightarrow$ loop corrections like 

increase $m^2_{h^0}$

$$\sim \frac{3}{4\pi^2}\cos^2\alpha\; y_t^2 m_t^2 \ln\left(\frac{m_{\tilde t_1} m_{\tilde t_2}}{m_t^2}\right)$$

- h behaves very much like SM higgs if A^0 heavy enough
- top squark mixing can enhance it further ($\tilde{t}$ from $\tilde{Q}_3$ & $\tilde{t}$ from $\tilde{\bar{t}}$)

but only $\lesssim$ 135 GeV (and that's pushing it)

can go on if add more stuff to MSSM

- Higgs: $h, H, A^0, H^\pm$
- neutralinos (fermions) $\tilde{W}^\pm, \tilde{G}, \tilde{H}_u^+, \tilde{H}_d^-$ mix charginos $\tilde{G}, \tilde{C}_1, \tilde{C}_2$

but $\tilde{B}^0, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0$: these can all mix after SM is broken by Higgs $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4$

- squarks
  sleptons

12 squark masses
9 slepton masses
4 neutralinos
2 charginos
1 gluino
4 $h, H, A^0, H^\pm$

---

32 masses
+ gravitino/goldstino

"Real" phenomenology

- all of them have SM couplings

  ⇒ they are naturally produced in $\left.\begin{array}{l} q\bar{q}, qq, gg \dots \\ e^+e^-, \dots \end{array}\right\}$ processes

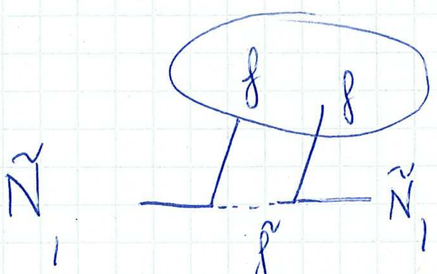
electroweak $q\bar{q} \to \tilde{C}_i^+ \tilde{C}_j, \tilde{N}_i \tilde{N}_j$.

so LEP in general mass $\gtrsim 100$ GeV
(LSP mass not too high) if $m_{LSP} \simeq m_{particle}$: difficult)

LHC : the signal must be visible
- enough missing energy
- enough visible energy

- decay chains towards LSP

  ⇒ characteristic multi $\left\{\begin{array}{l}\text{lepton} \\ \text{jet}\end{array}\right.$ signals



correlations

- 2 body decays possible but again typically a chain to get to LSP