large N_c

't Hooft, Nucl. Phys. B 72 (1974) 461 (large N)

't Hooft, Nucl. Phys. B 75 (1974) 461 (2D QCD large N)

Veneziano, Witten

E. Witten, Nucl. Phys. B 160 (1979) 57

Coleman, 1/N, Erice Lectures 1979 "classic"

SLAC-PUB-2484 (1980)

Manohar, large N_c QCD, 1998 Les Houches lectures "new view" of baryons

hep-ph/9802419

Notice I will neglect many sidelines in this of course.

Step 1 $N_c \to \infty$

Step 2 $SU(N_c) \to U(N_c)$

$$\sum_a T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right)$$

The extra $U(1)$ part is $1/N_c$ suppressed

Step 3: What to keep cst: in QCD

$$\mu \frac{dg_s}{d\mu} = - b_0 \frac{g_s^3}{16\pi^2} + \cdots \qquad b_0 = \frac{11}{3} N_c - \frac{2}{3} N_F$$

So: N_F becomes less important: gluons dominate

So we choose as limit: $g_s = \frac{g}{\sqrt{N_c}}$ and keep g constant

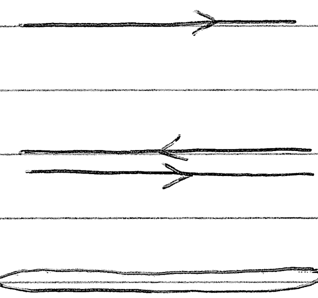
We now formally "add" and "subtract" a $U(1)$ gauge boson.

So A_μ becomes a full $N \times N$ hermitian matrix

Note the $U(1)$ does $\boxed{\text{not}}$ have self interactions, these always come with commutators.

All self interactions are ~~double~~ single traces so use a double line notation

$\bar{q}^{\alpha} A_{\alpha}^{\bar{\beta}} q_{\bar{\beta}}$

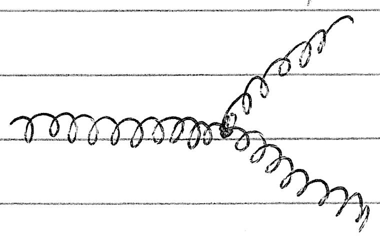


quark

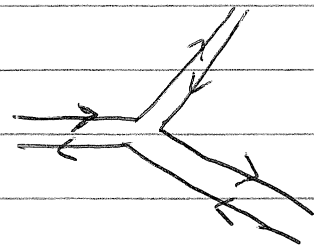
$U(N)$ gluon

$U(1)$ gluon (but no colour sum implied)

3 vertex $\mathrm{tr}\, \partial_\mu A_\nu [A_{\mu'} A_{\nu'}]$



$\Rightarrow$



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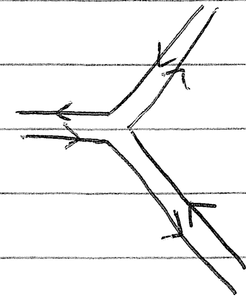
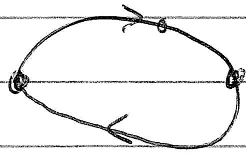
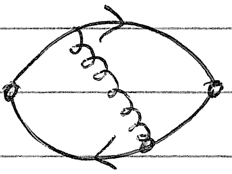
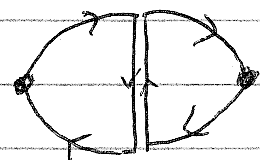


$\langle T(\bar{q}_1^{\alpha} q_{2\alpha}(x)\ \bar{q}_2^{\beta} q_{1\beta}(0)) \rangle$ and let's do an expansion for this

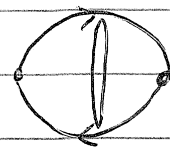


N_c

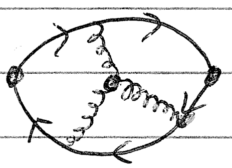


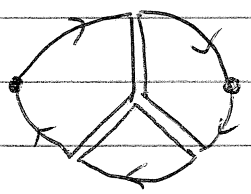


$\frac{g^2}{N_c} N_c^2 = N_c$

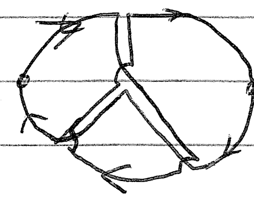


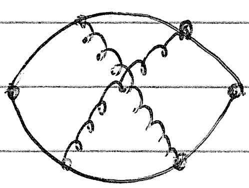
$\frac{g^2}{N_c} N_c$

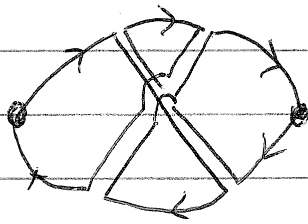




$\frac{g^4}{N_c^2} N_c^3 = N_c$



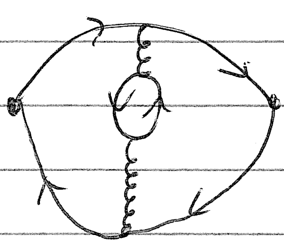


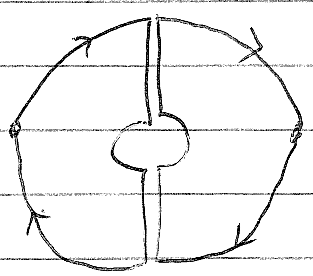


$\frac{g^4}{N_c^2} N_c = \frac{1}{N_c}$

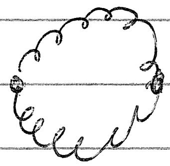
$\frac{g^4}{N_c^2} N_c = \frac{1}{N_c^2}$

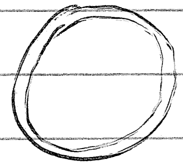
$\Rightarrow$ Planar graphs are leading





$\frac{g^4}{N_c^2} N_c^3 = 1$





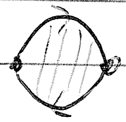
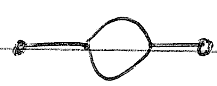
N_c^2

So a graph is

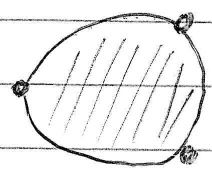
N^a : $a = 2 - \#\,\text{holes} - 2\#\,\text{handles}$

⇒ similar to string theory / dual model results.

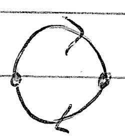
Confinement assumed

⇒  =  +  + ...

⇒ meson propagator $\mathcal{O}(1)$ meson operator is $\frac{1}{\sqrt{N_c}}\, \bar{q}_1^{\alpha}\, q_{2\alpha}$

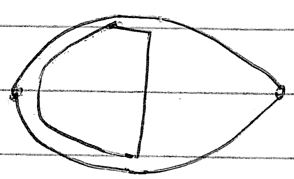
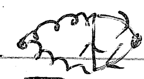
⇒ meson vertex  $\sim \frac{1}{(\sqrt{N_c})^3}\, N_c \sim \frac{1}{\sqrt{N_c}}$

goes to zero in large N_c limit

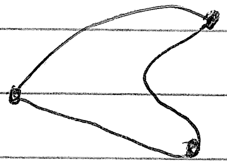
⇒  $\sim \ln q^2 \sim \sum_i \frac{f_i}{q^2 - m_i^2}$

⇒ ∞ number of mesons

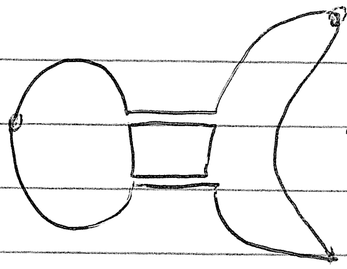
⇒ glueballs : ∞ number $\frac{1}{N_c}\, G G$

 $\frac{g^2 N_c^2}{N_c}\, \frac{1}{N_c \sqrt{N_c}}$  $\sim \frac{1}{\sqrt{N_c}}$ glueballs / quark meson mixing suppressed

Zweig's rule automatically follows

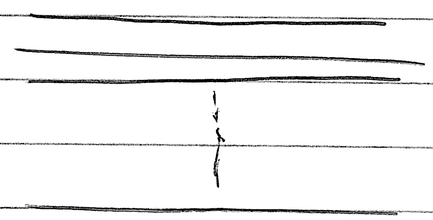
 $\sim \frac{1}{\sqrt{N_c}}$

but

 $\sim \frac{g^4}{N_c^2} N_c^2 \frac{1}{\sqrt{N_c}^3} \sim \frac{1}{N_c\sqrt{N_c}}$

Baryons are more complicated

N_c quarks $\sim \varepsilon^{\alpha_1 \dots \alpha_{N_c}} \, q_{\alpha_1} \cdots q_{\alpha_{N_c}}$



$$m_B \sim N_c m + \underbrace{\sum_{i \neq j} V(x_i - x_j)}_{N_c^2 \times \frac{g^2}{N_c}} + \dots$$

also order N_c

$\Rightarrow$ Hartree approximation is exact in large N limit (Witten)

Baryon Meson is $\sqrt{N_c}$

Baryon Baryon is N

BMB^* is $\sqrt{N}$

1 1 adj

1 2 1

1 2 2 1

⋮

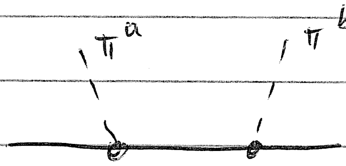
1 1

axial couplings $f_\pi \sim \sqrt{N}$

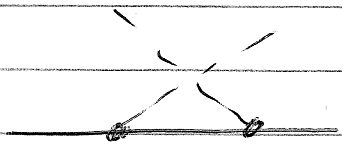
$\partial_i \langle B| \bar q \gamma^i \gamma_5 T^a q |B\rangle = g N \langle B| X^{ia} |B\rangle$

$\frac{\partial^\mu \pi^a}{f_\pi} \langle B| \bar q \gamma^\mu \gamma_5 T^a q |B\rangle$ is π baryon coupling

$\pi B \to \pi B$ is Order N

But 

is $O(N^2)$


is $O(N^2)$

$$\Rightarrow \quad [X^{ia}, X^{jb}] = 0$$

$$[J^i, X^{jb}] = i\, \varepsilon_{ijk} X^{kb}$$ usual spin

$$[T^a, X^{jb}] = i\, f_{abc} X^{jc}$$ usual flavour

$\Rightarrow$ almost an $SU(2N_F)$: J^i, T^a, G^{ia}

$$[G^{ia}, G^{jb}] = \frac{i}{2N_F} \varepsilon_{ijk} \delta_{ab} J^k + \frac{i}{4} f_{abc} \delta_{ij} T^c + \frac{i}{2} \varepsilon_{ijk} d_{abc} G^{kc}$$

so $X^{ia} = \lim_{N\to\infty} \left(\frac{G^{ia}}{N} \right)$

Lie algebra contraction — Gervais-Sakita — Dashen-Manohar

The unitarity triangle

This is an overview of where the different elements of the Cabibbo Kobayashi-Maskawa mixing matrix are determined.
Discussions of the difficulties that appear in each of the various processes and the errors appear later when we talk about semileptonic and non-leptonic decays.
As shown in section 1 we have a mixing matrix with a nontrivial phase. This can be seen in a parametrization invariant way by looking at the Jarlskog determinant

$$\det[M_u, M_D] = s_1^2 c_1 s_2 c_2 c_3 s_3 c_3 s_\delta \prod_{\substack{\text{all}\\ \text{pairs}\\ i,j}} (m_{u_i} - m_{u_j})(m_{d_i} - m_{d_j})$$

so no CP violation if:

$s_\delta = 0$, $\delta_i = 0$ or $\frac{\pi}{2}$ for one i

or 2 up or 2 down masses are equal (in the latter case we cannot determine one of the mixing angles => can be set $= 0$ or $\frac{\pi}{2}$).

For more than 3 generations it is sufficient if this condition is satisfied for any one of the 3×3 submixings.

V_{ud} : $\pi^+ \to \pi^0 e^+ \nu$
β decay of nuclei * } 0.9736 ± 0.0010

V_{us} : hyperon β decay
$K \to \pi l \nu$ * } 0.2205 ± 0.0018

This gives a first constraint from Unitarity :

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\Rightarrow \quad |V_{ub}| \lesssim .055$$

V_{cd} : charm production in neutrino scattering : .224 ± .017
unitarity ⇒ $|V_{us}|$

V_{cs} : $c \to s$ decays or charm production of the strange sea in neutrinoscattering (1.01 ± 0.18)

V_{ub} : $b \to u$ charmless B decays
$|V_{ub}/V_{cb}| \simeq .08 \pm 0.02$

V_{cb} : B lifetime similar unitarity bound to $|V_{ub}|$; notice that in this case the bound is almost saturated.
$|V_{cb}| \simeq 0.041 \pm 0.003$

V_{tb} : in principle in top decays $t \to b W^+$

on V_{td} and V_{ts} in the near future there will only be indirect constraints

So how does unitarity work:

V_{ud}, V_{us} & $|V_{ub}|$, V_{cb} experimentally well bound
⇒ this determines essentially V_{cd} and V_{cs} and V_{tb}
$|V_{td}|$ and V_{tb} also constraint.

Experimentally only the phase δ is little determined: need CP violating observables.

in Wolfenstein : λ from $K \to \pi l \nu$

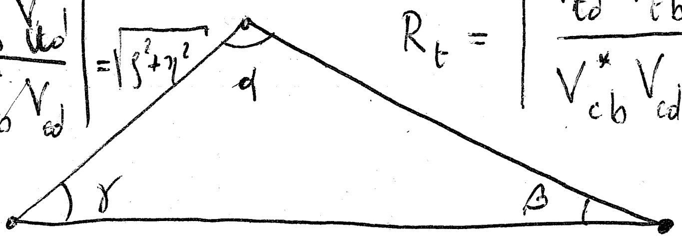
A from $b \to c$

$\sqrt{\rho^2+\eta^2}$ from $b \to u$

$$\underbrace{V_{ud}^*}_{1-\lambda^2} V_{ub} + \underbrace{V_{cd}^*}_{-\lambda} V_{cb} + \underbrace{V_{td}^*}_{1} V_{tb} = 0 \qquad \text{is the unitarity triangle.} \qquad (*)$$

or a sum of 3 complex numbers vanishes => the form a closed triangle in the complex plane

$$R_b = \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right| = \sqrt{\rho^2+\eta^2} \qquad R_t = \left| \frac{V_{td}^* V_{tb}}{V_{cb}^* V_{cd}} \right| = \sqrt{(1-\rho)^2+\eta^2}$$



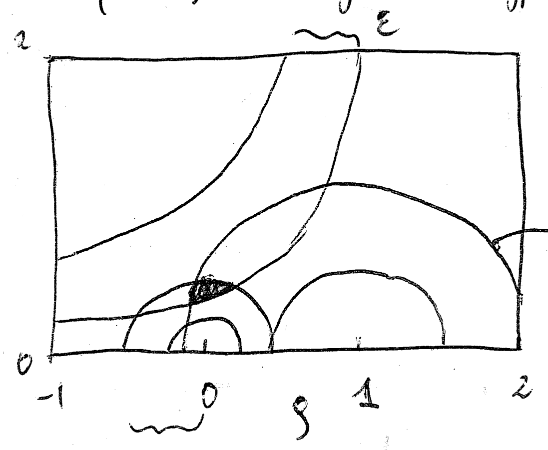
(0,0) (1,0)

notice that all terms in (*) are order λ^3.

The length of the sides can in principle be measured from non CP. violating quantities.

a 3rd constraint comes from any CP. violating quantity (so far only $\varepsilon \neq 0$ ...)

$\mathrm{Im}\, |V_{td}|^2 \propto \eta(1-\rho)$ so gives a hyperbolic constraint



$m_t = 174$

R_t : B_0-$\bar{B}_0$ mixing

$|V_{ub}/V_{cb}| \sim R_b$

Effective field theory / Matching

A fundamental field theory
- renormalizable: to have predictions at all orders
- well defined
- expansions are typically in small coupling constants

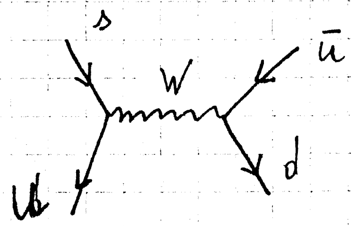
An effective field theory:
- can be nonrenormalizable
- calculational tool and/or because we cannot do better
- degrees of freedom can be very different from underlying theory
- folktheorem by Weinberg
- field redefinitions can make theories look very different
- expansions can be in other quantities than coupling constants
  power counting
- Using dimensions in constructing effective Lagrangians

Matching:
- integrating out (often in principle, not in practice)
- best way: calculate observables in both theories
  put equal $\Rightarrow$ match parameters in both theories
  (remember limits here!)
- examples:
  - QCD $n_f \leftrightarrow n_f - 1$ quarks: thresholds
  - W-exchange versus Fermi interaction
    (even at one loop)
  - Linear versus nonlinear sigma model
  - QED $\leftrightarrow$ HQET

The meaning of loops and renormalization in effective theories

In this part I will first give a heuristic description of the meaning of loops in nonrenormalizable theories. I will explain this difference using the example off an effective theory with quarks only and an underlying theory with quarks and W-bosons. Since here I want to show how to deal with the infinite parts I will not explicitly calculate diagrams nor introduce complications like GIM-mechanism, real mixing angles, QCD corrections, ...

The decay of an s-quark is given by



(1)

This is given by the amplitude:

$$\left[\bar{s}\gamma_\mu(1-\gamma_5)u\right]\left[\bar{u}\gamma^\mu(1-\gamma_5)d\right] \qquad \frac{i}{p^2_{\bar{u}d}-m^2_W} \qquad \frac{g_W^2}{8} \tag{2}$$

At low energies we describe this by a Lagrangian which only has quarks. This we do via the Fermi interaction of the type

$$\mathcal{L}^{(1)}_{eff} = \frac{G_F}{\sqrt{2}}\left[\bar{s}\gamma_\mu(1-\gamma_5)u\right]\left[\bar{u}\gamma^\mu(1-\gamma_5)d\right] \tag{3}$$

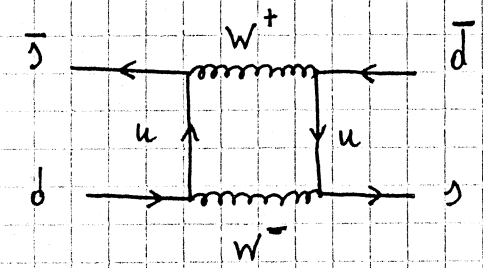
Comparing the amplitude that follows from (3) with the one that follows from diagram (1) at <u>low</u> energies we obtain

$$G_F = \frac{-g_W^2}{4\sqrt{2}\,m_W^2} \tag{4}$$

So: at low energies (1) and (3) are the same and the parameters in $\mathcal{L}_{eff}$ can be determined by calculating the process both in the underlying theory and in the effective theory and setting both results to be equal. This gave (4) as the result.

Let us now do the same for $K^0 \bar{K}^0$ mixing. This is the process where

$\bar{s}\, d \rightarrow s\, \bar{d}$ and proceeds via the diagram

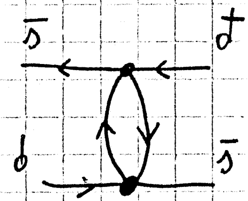


+ crossed (5)

At low momenta this amplitude is

$$A = b \int \frac{d^d p}{(2\pi)^d} \frac{p^2}{(p^2)^2 \, (p^2 - m_W^2)^2} + \text{crossed} = i\, c\, G_F^2 \qquad (6)$$

so the results is finite and well defined. Now what happens if we try to calculate this process using our effective theory (3). We then have the Feynman diagrams



+ crossed (7)

which gives an amplitude:

$$A = b' \int \frac{d^d p}{(2\pi)^d} \frac{p^2}{(p^2)^2} + \text{crossed} = i(c' \Lambda^2 + c'')\, G_F^2 \qquad (8)$$

This integral is quadratically divergent! So how can we describe $K^0 \bar{K^0}$ mixing in our effective theory? (Λ is the cut-off)

The solution is that we forgot the possibility of more terms in our effective Lagrangian. We restricted the Lagrangian to terms with only quarks so we can add to (3):

$$\mathcal{L}^{(2)}_{eff} = \mathcal{L}^{(1)}_{eff} + a \, [\bar{s} \gamma_\mu (1-\gamma_5) d][\bar{s} \gamma^\mu (1-\gamma_5) d] \qquad (9)$$

So now the amplitude in our effective theory is:

$$i\left(a + (c'\Lambda^2 + c'')G_F^2\right) = i\, c\, G_F^2 \qquad (10)$$

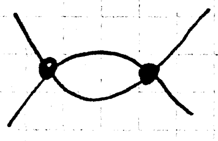
The equality follows because our effective theory should reproduce the more fundamental underlying theory.
So we now set $a = a_{inf} + a_r$ with $a_{inf} = -c'\Lambda^2$ so we obtain

$$a_r + c'' G_F^2 = c\, G_F^2 \qquad (11)$$

So what have we done:

- we included <u>all</u> allowed terms in our effective Lagrangian
- the infinite parts that occur in the loop diagrams are absorbed into the coefficients of the new terms, these occur at higher orders in G_F^2.
- From our effective theory alone we could only determine K^0-$\bar{K}^0$ mixing up to the free parameter a_r.

In the remainder of this part I will prove that a similar procedure can be carried out for the case of chiral perturbation theory. There we will not do an expansion in G_F but do an expansion in energies, momenta and masses. These we all denote by p.
As shown in the previous section the lowest order is order p^2. So what is the next order. We only have mesons so Lorentz invariance requires the next order to be p^4 in the effective Lagrangian. What happens now with one loop diagrams? Take

 (12)