6 Symmetries in the QCD lagrangian

As could be seen before the strong interaction is very often the main problem even if we wish to determine electroweak parameters.
In the absence of an easy method of calculating one resorts to the use of symmetries and approximate models. In these lectures mainly the symmetry aspects will be talked about. A review of approximate models can be found in the book by Donoghue.
A direct attack on the problem is to use lattice field theory. This is covered by J. Smit here.
The QCD lagrangian is extremely simple ($\theta = 0$)

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + i \sum_i \bar{q}_i \gamma^\mu D_\mu q_i - m_i \bar{q}_i q_i$$

We now add to this

$$-\bar{q} v_\mu \gamma^\mu q - \bar{q} a_\mu \gamma^\mu \gamma_5 - \bar{q} s q + \bar{q} i p \gamma_5 q$$

Classically this has a

$$\mathbb{R} \times U(6)_L \times U(6)_R \text{ symmetry} \qquad (\text{include } m_i \text{ in } \bar{s} = s + (m_i))$$

$$\mathbb{R}: \quad x \to \frac{x}{\lambda}, \quad G^a_\mu, v_\mu, a_\mu \to \lambda(\ldots)$$

$$q \to \lambda^{3/2} q$$

$$s, p \to \lambda^{\bullet} q$$

broken by the scale anomaly

$$q_{L,R} \to g_{L,R}\, q_{L,R}$$

$$v_\mu \pm a_\mu \to g_{L,R} (v_\mu \pm a_\mu) g_{L,R}^\dagger + i \partial_\mu g_{L,R}\, g_{L,R}^\dagger$$

$$s + ip \to g_L (s + ip) g_R^\dagger$$

Now $m_c, m_b, m_t \gg \Lambda_{QCD}$ so choosing these as zero is not a good starting point (but see further)
So we use

$$U(3)_L \times U(3)_R = U(1)_V \times U(1)_A \times SU(3)_L \times SU(3)_R$$

$U(1)_A$ ↳ broken by anomalies : coupled to instantons

$U(1)_V$ ↳ baryon number.

So the "useful" symmetry is $SU(3)_L \times SU(3)_R$: its consequences are embedded in chiral perturbation theory (CHPT).
There is in fact also a symmetry that is useful for the heavy quarks : heavy quark effective theory (HQET).
This symmetry is a little more tricky to unearth ; for one quark we have

$$\mathcal{L}_1 = \sum_j i \bar{Q}_j \gamma^\mu \partial_\mu Q_j - M_j \bar{Q}_j Q_j - \bar{Q}\, g_s \not{G}\, Q$$

we now take an almost on-shell quark with momentum p and write

$$p_j = M v + k_j$$

k we assume to be small and we write

$$Q_j = e^{-iM_j v \cdot x} h_j \qquad \text{with} \qquad \not{v}\, h = 0$$

~~Alternatively~~ We put this into $\mathcal{L}_1$ and obtain

$$\mathcal{L} = \sum_j \bar{h}_j\, i v \cdot D\, h_j$$

h_j has 2 free Dirac components and the $\sum_j$ is identical $\Rightarrow$

there is an $SU(2N)$ extra symmetry with N the number of heavy flavours. Notice that all quarks should move with about the same velocity v (velocity selectionrule) in order to use this symmetry.

This is the HQET spin flavour symmetry. Notice also that this is a theory with a fixed number of heavy quarks. There is no pair creation of heavy quarks in the heavy quark limit.
A more formal derivation is

$$Q = e^{-iM v\cdot x}(h + X)$$

where $\begin{cases} \not{v} h = h \\ \not{v} X = -X \end{cases}$

we then determine X as a function of h via the equations of motion. (Georgi, Isgur, Wise). But notice v_i ~~and~~ can be different for different quarks.
An alternative derivation is via a series of Foldy-Wouthuysen transformations (Körner, Pirjol et al.)
The spin symmetry is included since the rest-frame of the heavy quark is a preferred frame so the Lorentz symmetry is ~~~~ explicitly broken.

From here on we will use the method of effective Lagrangians very extensively. In its simplest form it sounds almost trivial:

Use a field theory where the fields are the relevant states for the energyscale under considerations.

In practice we cannot always calculate things so easily and we can only use the underlying theory in its symmetry aspects. Sometimes we can explicitly calculate: this is usually called matching and has to be done at the order that we are working.
In practice we can of course only take a limited number of terms.
So:
- construct an effective lagrangian to the desired order with the coefficients constrained by the underlying symmetries
- if possible: match the coefficients to the underlying theory

Sometimes there are additional matching conditions that can be applied. 2 examples are

- 't Hoofts anomaly conditions: the effective theory should give rise to the same anomaly as the original theory.
- reparametrization invariance: in the HQET case we introduced an arbitrary parameter v. Shifts in this parameter should not change the results

$$\mathcal{L} = \bar{h}\, i(v\cdot D)h + \frac{1}{2m_Q}\,\bar{h}\left\{\alpha (iD)^2 + \beta\,(v\cdot D)^2\right\}h$$

the term β is proportional to $(v\cdot D)\,h = 0$ because of the lowest order equations of motion (or equivalently can be removed via a field redefinition)

now $v \to v+\varepsilon$ should be an invariance

$$v^2 = (v+\varepsilon)^2 = 1 \quad \Rightarrow \quad v\cdot\varepsilon = 0$$

$(\not{v}\, h) = h$ requires

$$(\not{v}+\not{\varepsilon})\, h + \not{v}\,\delta h + \cancel{\not\varepsilon\,\delta h} = h + \delta h$$

or $\quad \not{\varepsilon}\, h + \not{v}\,\delta h = \delta h$

and δh should have been in X $\Rightarrow$ $\not{v}\,\delta h = -\delta h$

or $\quad \delta h = \dfrac{\not{\varepsilon}}{2}\, h$

now $v \to v+\varepsilon$ then $\quad Mv + k = M(v+\varepsilon) + (k - M\varepsilon)$

and $\mathcal{L}$ invariant requires $\quad \alpha = 1.$

$$\left\{\begin{array}{l} D \to -ik \qquad \Rightarrow \quad -ik + iM\varepsilon \\ v \to v+\varepsilon \end{array}\right.$$

$$v\cdot D + \frac{\alpha}{2M}\, i D^2 \to -i\,i\,(v+\varepsilon)\cdot(k - M\varepsilon) + \frac{\alpha}{2M}\,(-1)\,(k^2 - 2\,k\cdot\varepsilon M)$$

$$= +1\, v\cdot k - \frac{\alpha}{2M}\,k^2 + (k\cdot\varepsilon - \alpha\, k\cdot\varepsilon)$$

$$\Rightarrow \quad \alpha \equiv 1$$

8. Semileptonic decays : heavy quarks

Let us first look at exclusive decays:

$B \to D e\nu$

$B \to D^* e\nu$

These are determined by $|V_{cb}|$ and the $\bar{c}\gamma_\mu(1+\gamma_5)b$ in between B and $D^{(*)}$ states. These are in general described by 6 formfactors

$$\langle D | \bar{c}\gamma_\mu b | B\rangle = f_+ (p+p')_\mu + f_- (p-p')_\mu \qquad (**)$$

$$\langle D^* | \bar{c}\gamma_\mu b | B\rangle = i g \, \varepsilon_{\mu\nu\alpha\beta} \, \varepsilon^{*\nu} (p+p')^\alpha (p-p')^\beta$$

$$\langle D^* | \bar{c}\gamma_\mu\gamma_5 b | B\rangle = f_1 \varepsilon^*_\mu + \varepsilon^* \cdot (p-p') \left[f_2 (p+p')_\mu + f_3 q_\mu \right]$$

$$q = p - p'$$

There are 6 formfactors involved. We also have

$$\langle B | \bar{b}\gamma_\mu b | B\rangle = f_B (p+p')_\mu \qquad (*)$$

$$\langle D | \bar{c}\gamma_\mu c | D\rangle = f_D (p+p')_\mu$$

now take $(*)$ and $(**)$ with $p = M_B v$ and $p' = M_{D(B)} v'$

$$q^2 = (p-p')^2 = M_B^2 + M_D^2 - 2 M_B M_D \, v\cdot v'$$

$$= (M_B - M_D)^2 + 2 M_B M_D (1 - v\cdot v') = t_m + 2 M_B M_D (1 - v\cdot v')$$

in the heavy quark limit

$$\frac{\langle D | \bar{c}\gamma_i b | B\rangle}{\sqrt{4 m_B m_D}} = \frac{\langle B | \bar{b}\gamma_i b | B\rangle}{2 m_B} \quad \to \text{relativistic versus nonrelativistic normalization}$$

or

$$\frac{f_B[x]}{2m_B} 2 m_B (v+v')_i = \frac{f_D[y]}{2m_D} 2 m_D (v+v')_i = \frac{(f_+ + f_-) M_B v + (f_+ - f_-) M_D}{\sqrt{4 m_B m_D}}$$

$$x = 2 M_B^2 (1 - v\cdot v')$$

$$y = 2 M_D^2 (1 - v\cdot v')$$

$$z = t_m + 2 M_D M_B (1 - v\cdot v')$$

so we obtain relations between all the formfactors if expressed as a function of $v \cdot v'$

$$f_B(v \cdot v') = f_D(v \cdot v') = \frac{\sqrt{4 m_B m_D}}{m_B + m_D} f_+ = -\frac{\sqrt{4 m_D m_B}}{-m_D + m_B} f_- = \xi(v \cdot v')$$

ξ is known as the Isgur Wise function.

g, f_1, f_2 and f_3 can also be derived similarly using the spin part of the spinflavour symmetry.

We also know $\xi(1) = 1$ (flavour symmetry)

(The spin symmetry is given by

$$e_b^2 = -1$$

$$e_a \cdot e_b = -\delta_{ab}$$

$$v \cdot e_a = 0$$

$$S^a = \frac{i}{8} \sum_{b,c} \varepsilon^{abc} [\not{e}_b, \not{e}_c]$$ generates the spin symmetry)

for $v = (1, \vec{0})$, $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$ $\quad S^a = \frac{1}{i} \begin{pmatrix} \sigma^a & 0 \\ 0 & \sigma^a \end{pmatrix}$

and everything becomes Pauli spinors)

In fact $\xi(1) = 1$ is the main underlying part in trying to determine V_{cb}.

The experimental problem is to extrapolate to that point, one measures

$$\frac{\langle D^* | \bar{c} \gamma_\mu b | B \rangle}{\sqrt{m_B m_{D^*}}} = 2 \eta_A \varepsilon^*_\mu$$

at $v \cdot v' = 0$, one typically extrapolates using a linear dependence on $v \cdot v'$

η_A is known to $O(\alpha_s^2 \beta_{(0)})$: $\eta_A = 1 - 0.03 - 0.005$

The size of the $\frac{1}{m_c}, \frac{1}{m_b}$ contributions is at present a controversial issue.

Inclusive decays: $B \to X_{c,u} e \bar{\nu}$

Here is where most theoretical progress has been made lately.
This can be done using the operator product expansion analogous to the QCD treatment of deep inelastic scattering.

$$W^{\mu\nu}_{c,u} = (2\pi)^3 \sum_X \delta^4(P_B - q - P_X) \langle B | J^{\mu +}_{c,u} | X \rangle \langle X | J_{\mu c,u} | B \rangle$$

$$= -g^{\mu\nu} W_1 + v^\mu v^\nu W_2 - i \varepsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta W_3 + q^\mu q^\nu W_4 + (q^\mu v^\nu + q^\nu v^\mu) W_5$$

$$T^{\mu\nu} = -i \int d^4x \, e^{-iq\cdot x} \langle B | T(J^{\mu +}_{c,u}(x) J^\nu_{c,u}(0)) | B \rangle$$

can be expanded similarly in T_i

and

$$\text{Im}\, T_i = -\pi W_i$$

One then performs an operator product expansion for $T^{\mu\nu}$ and then goes to HQET. This parametrizes all unknowns in matrix elements of the type $\langle \tilde{B} | O_i | B \rangle$ which are independent of q^2 (or "$v \cdot v'$")

The leading operators are:

$\bar{b} \gamma_\mu b$

$\bar{h}_v \frac{iD^2}{2m_b^2} h_v$: only unknown, the kinetic energy of the b-quark

$\langle B | \bar{h}_v \frac{g_s G^{\mu\nu} \sigma_{\mu\nu}}{4 m_b^2} h_v | B \rangle$ also in $B^* - B$ splitting

In the endpoint region there are problems with singular terms. Their origin is that HQ treats $b \to X e \nu$ but in $B \to X e \nu$ are higher electron energies possible.

$$\frac{1}{\Gamma_0}\frac{d\Gamma_0}{dy} \sim \sum_{n=0} \delta^{(n)}(1-y)\,\varepsilon^{n+1} + \text{nonsingular} \qquad y = \frac{2E_e}{m_b}$$

$\downarrow$ n'th derivative of the δ function

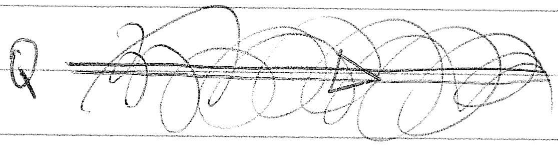
this is singular so one needs to smear over a region of order ε, but the number of unknowns remains to high.
However the same singularities appear in $B \to X_s \gamma$ so it can be studied. At present one relies on models.

This leads to $\left|\frac{V_{ub}}{V_{cb}}\right| \simeq 0.08 \pm 0.01$ but error is model dependent.

When we looked at the heavy quark limit in semileptonic decays we derived the relations at fixed velocity, explain why?

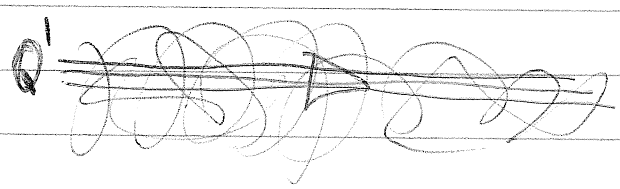
For heavy quarks what happens is that in these systems the corrections away from the infinitly heavy quark limit are of order $\frac{p}{m_Q}$.

What we need is that the system after exchanging one heavy quark for another one is identical as far as the rest of the system is concerned.



$$p_T \simeq m_Q v + \text{rest}$$

$$\Updownarrow$$



$$p_T' = m_{Q'} v + \text{rest}$$

If we had kept e.g. $p_T = p_T'$ the internal diagrams would be very different in both cases and the corrections internally would be of order $\frac{m_Q - m_{Q'}}{m_{Q'}}$ which is not necessarily small.

By performing the exchange instead at fixed velocity the difference between the two cases is of order $\frac{\text{rest}}{m_Q}$ and $\frac{\text{rest}}{m_{Q'}}$

The linear sigma model

Rather than first giving general proof let us treat the linear sigma model as an example.
This has an

$$O(4) = SU(2) \times SU(2)$$

symmetry.
I will use the $SU(2) \times SU(2)$ notation

$$\Sigma = \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma + \pi^3) & \frac{\pi^1 + i\pi^2}{\sqrt{2}} \\ \frac{\pi^1 - i\pi^2}{\sqrt{2}} & \frac{1}{\sqrt{2}}(\sigma - \pi^3) \end{pmatrix}$$

$$\Sigma^\dagger = \Sigma$$

$$\mathcal{L}_{l\sigma} = \frac{1}{2}\, \mathrm{tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) - \lambda \left[\mathrm{tr}\left(\Sigma\Sigma^\dagger - v^2\right)\right]^2$$

this is renormalisable and

$$\mathcal{L}_{l\sigma} = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi^i \partial^\mu \pi^i\right) + \ldots$$

(note: summed indices $i = 1, .., 3$ here)

$\mathcal{L}_{l\sigma}$ has an $SU(2) \times SU(2)$ symmetry

$$\Sigma \longrightarrow g_R \Sigma g_L^\dagger$$

In order to start further we need to find the vacuum

- the kinetic term and the potential are both always positive
  $\Rightarrow$ both zero is the best solution

or $\sigma^2 + \pi^i \pi^i = v^2$

Notice that we have a lot of possibilities here: in effect $O(3)$ or $SU(2)$ ways of describing the vacuum exist.

The symmetry $SU(2) \times SU(2)$ actually rotates these different states into each other.
So we choose the canonical choice.

$$\sigma = v \quad , \pi^i = 0$$

Or

$$\langle \Sigma \rangle = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and the symmetry operations with $g_R^\dagger = g_L$ do not change $\langle \Sigma \rangle$

let us now look at the particle spectrum:

$$\sigma = v + \tilde{\sigma}$$

$$\Rightarrow \quad \mathcal{L} = \frac{1}{2} \partial_\mu \tilde{\sigma} \partial^\mu \tilde{\sigma} + \partial_\mu \pi^i \partial^\mu \pi^i$$

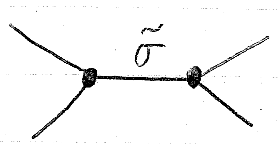
$$- \lambda\, 4 v^2 \tilde{\sigma}^2 - \lambda\, 4 v\, \tilde{\sigma}\, \pi^i \pi^i - \lambda\, \pi^i \pi^i \pi^j \pi^j - \lambda\, \tilde{\sigma}^4$$

$$- 4\, \lambda v\, \tilde{\sigma}^3 - 2 \lambda\, \tilde{\sigma}^2 \pi^i \pi^i$$

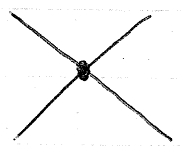
So one massive σ particle and 3 massless pions.

$$m_\sigma^2 = 8 \lambda v^2$$

Let us now calculate $\pi^1 \pi^1 \to \pi^2 \pi^2$ scattering



$$= \frac{i^3}{s - m_\sigma^2} \qquad +16 v^2 \lambda)^2 \qquad 4$$



$$= \quad i(-\lambda) \quad 8$$

Exercise

Let's now look at scattering for small s:

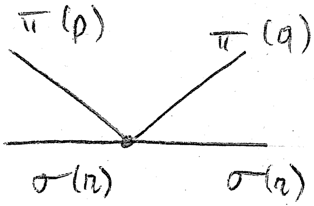
$$A = i \, 8 \lambda \left(-1 - \frac{8 v^2 \lambda}{s - m_\sigma^2} \right)$$

$$= i \, 8 \lambda \left(\frac{m_\sigma^2}{m_\sigma^2 - s} - 1 \right)$$

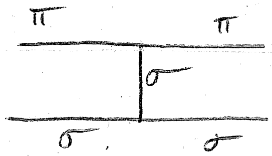
$$= i \, 8 \lambda \left(\frac{s}{m_\sigma^2} + \frac{s^2}{m_\sigma^4} + \cdots \right) = \frac{i s}{v^2} + \cdots$$

So even if it looks like a nonderivative coupling $\pi\pi$ scattering vanishes at $s = 0$!

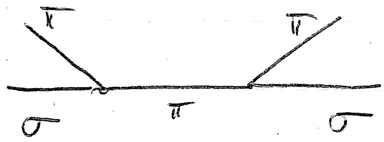
This is a more general feature of this lagrangian: look at $\pi^1 \tilde{\sigma}$ scattering



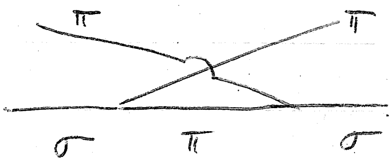
$$= i\,(-\lambda 2)\,4$$



$$= i^3 \frac{1}{(p-q)^2 - m_\sigma^2}\,(-4\lambda v)\,6\,(-4\lambda v)\,2$$



$$= i^3 \frac{1}{(r+p)^2}\,(-\lambda 4 v)^2\,4$$



$$= i^3 \frac{1}{(r-q)^2}\,(-\lambda 4 v)^2\,4$$

$r^2 = m_\sigma^2$ remember

$$A = +i 8 \lambda \left(-1 + \frac{3 \cdot 8 \lambda v^2}{m_\sigma^2 - (p-q)^2} - \frac{8\lambda v^2}{m_\sigma^2 + 2 p\cdot r + p^2} - \frac{8 \lambda v^2}{m_\sigma^2 - 2 q\cdot r + q^2}\right)$$

$$= i 64 \lambda^2 v^2 \left(+ \frac{(p-q)^2}{m_\sigma^2} + \frac{2p\cdot r + p^2}{m_\sigma^2} - \frac{4 (p\cdot r)^2}{m_\sigma^4} + \frac{-2 q\cdot r + q^2}{m_\sigma^2} - \frac{4 (q\cdot r)^2}{m_\sigma^4}\right)$$

$$= i 64 \lambda^2 v^2 \left(\frac{2 r\cdot(p-q)}{m_\sigma^2} + \frac{3(p-q)^2 + p^2 + q^2 - 4\left((p\cdot r)^2 + (q\cdot r)^2\right)/m_\sigma^2}{m_\sigma^2} + \dots \right)$$

σ on-shell: $\quad p + r = q + r'$

or $\quad p - q + r = r'$

square

$$\Rightarrow \quad 2(p-q)\cdot r + (p-q)^2 = 0$$

again it vanishes at zero pion momentum and only starts at momentum p, q of order 2!

The nonlinear σ model

So from Goldstone's Theorem we have 3(8) Goldstone Bosons for the case of $SU(N) \times SU(N) \to SU(N)$ for $N = 2, 3$
We treated the linear sigma model before, this is a model with $N = 2$.
So let's try to construct an effective Lagrangian with only pions.

$$U \to g_R U g_L^+$$

$$U^+ U = 1$$
$$\det U = 1$$

U describes an $SU(N)$ matrix $\simeq SU(N) \times SU(N) / SU(N)$
so it has the correct structure from the symmetry point of view.

No invariant Lagrangian without derivatives exist.
With 2 derivatives there is only

$$\mathcal{L} = \frac{F^2}{4} \operatorname{tr} \partial_\mu U \partial^\mu U^+$$

$$U = \exp\left[\frac{i}{F} \underbrace{\left(\pi^i \tau^i \right)}_{M} \right] \qquad (\text{or } \pi^a \lambda^a \text{ for 3 flavours})$$

So expanding U:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \pi^i \partial^\mu \pi^i + \frac{1}{24F^2} \operatorname{tr}\left(\partial_\mu M M \partial^\mu M M - \partial_\mu M \partial^\mu M M M \right)$$

$$= \frac{1}{2} \left(\partial_\mu \pi^i \partial^\mu \pi^i \right) + \frac{2}{24F^2} \Big[4 \pi^1 \pi^2 \partial_\mu \pi^1 \partial^\mu \pi^2 - 2 \pi^{1^2} \partial_\mu \pi^2 \partial^\mu \pi^2$$
$$- 2 \pi^{2^2} \partial_\mu \pi^1 \partial^\mu \pi^1$$

a $\pi^1\pi^1 \to \pi^2\pi^2$ is

$$\frac{i2}{24F^2}\left[4(p_1+p_2)\cdot(p_4'+p_2') + 8\, p_1\cdot p_2 + 8\, p_1'\cdot p_2'\right]$$

$$= \frac{i2}{24F^2}\left[+8\,p_1\cdot p_2 + 8\,p_1'\cdot p_2' + 4s\right]$$

$$= \frac{i2}{24F^2}\left[12s - 4(p_1^2+p_2^2+p_3^2+p_4^2)\right]$$

$$= \frac{i}{F^2}\left[s - \frac{1}{3}(p_1^2+p_2^2+p_3^2+p_4^2)\right]$$

If we identify v & F we see that the scattering amplitude agrees on-shell.

<u>Lessons</u>

1) only physical quantities agree : there are different off-shell parametrizations possible.

2) If we know the underlying "theory" we can calculate the parameters of the effective model

3) The fact that it vanishes at $s=0$ is immediately obvious: the power counting is explicit.

<u>Note</u> : ① I could have done this directly by following the parametrization

$$\Sigma_1 = \frac{\sigma}{\sqrt{2}}\, U$$

instead but we wouldn't have learnt as much.

② I could have done something similar for $\pi\sigma$ scattering : put off till later

The Higgs sector as a nonlinear sigma model

The Higgs sector of the standard model is given by

$$\mathcal{L} = \frac{1}{2}\left(D_\mu \phi\right)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda\left(\phi^\dagger \phi\right)^2 \tag{1}$$

with $\mu^2 < 0$ and the complex ϕ field gets a vacuum expectation value

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

using $$D_\mu \phi = \partial_\mu \phi - i\frac{g}{\sqrt{2}}\begin{pmatrix} \frac{W^0_\mu}{\sqrt{2}} & W^+_\mu \\ W^-_\mu & \frac{-W^0_\mu}{\sqrt{2}} \end{pmatrix} - i\frac{g'}{2} B_\mu$$

this leads to a mass-term for the W and B as

$$\mathcal{L}_{mass} = \frac{1}{2}\left(\frac{gv}{2}\right)^2 W^+_\mu W^{-\mu} + \frac{1}{2}\left(\frac{g'v}{\sqrt{2}} B_\mu - \frac{gv}{\sqrt{2}} W^0_\mu\right)^2 \tag{2}$$

which leads to the usual masses for the W and the Z^0 and a massless photon.

If we now assume that the Higgs sector consist of Goldstone Bosons coming from an underlying theory we replace (1) by

$$\mathcal{L}_{TC} = \frac{F^2}{4} \operatorname{tr}\left(D_\mu U D^\mu U^\dagger\right) \tag{3}$$

with under $SU(2)_L \times SU(2)_R$ $\quad U \to g_L U g_R^\dagger$

and $$D_\mu U = \left[\partial_\mu U - \frac{ig}{\sqrt{2}}\begin{pmatrix} W^0_\mu/\sqrt{2} & W^+_\mu \\ W^-_\mu & -W^0_\mu/\sqrt{2} \end{pmatrix}\right] U + U \begin{pmatrix} +\frac{ig'}{\sqrt{2}} B_\mu & 0 \\ 0 & -i\frac{g'}{\sqrt{2}} B_\mu \end{pmatrix}$$

This in turn leads to a mass term (set $U = 1$ unitary gauge, this can always be done via an $SU(2)_L$ gauge transformation $U^\dagger = g_L$)

$$\mathcal{L}_{TC} = \frac{g^2 F^2}{4} W^+_\mu W^{-\mu} + \frac{F^2}{2}\left(\frac{g'}{\sqrt{2}} B_\mu - \frac{g}{\sqrt{2}} W^0_\mu\right)^2 \qquad (4)$$

which leads to the same mass term as (2) if we identify $\frac{v}{\sqrt{2}} = F$

But this is to $O(p^2)$ then the only remnant of the Higgs sector.

Some remarks:

A) Of course at $O(p^4)$ there will be other terms which do leave other interactions:

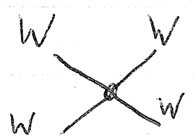
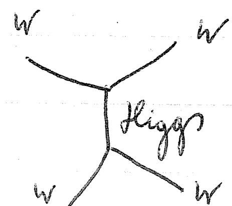
E.g. the L_{10} term: $\mathcal{L} = L_{10}\, \mathrm{tr}\, \ell_{\mu\nu} U r^{\mu\nu} U^\dagger \qquad (5)$

contains terms like $B_{\mu\nu} W^{0\mu\nu}$

removing these when you diagonalise the kinetic term changes the relation $m_W^2 = \cos^2\theta_W\, m_Z^2$

In fact the present LEP measurements rule out a deviation from this relation by as much as would be expected from (5) if we scale up the measured value of L_{10} from F_π to $F = \frac{v}{\sqrt{2}}$. This is the underlying argument when it is said that LEP has ruled out simple technicolour.

B) The term proportional to L_9 leads to observable consequences at LEP-II, usually labelled as a deviation from the expected dipole and quadrupole moment of the W.

C) At high energies  is cancelled by . This is now not present anymore so the W becomes strongly interacting at high energies.