{ Flavianet report. arXiv: 0801.1817
{ add lepton universality from $\pi \to \mu\nu / e\nu$ $\quad K \to \frac{\mu\nu}{e\nu}$

let us now go systematically through the decays

(I) $\pi_{\ell 2}$: $\begin{cases} \pi \to \mu\nu \\ \pi \to e\nu \end{cases}$: $|V_{ud}|$, F_π

$K_{\ell 2}$ $\quad K \to \mu\nu$ $\quad |V_{us}|$, F_K

$\quad K \to e\nu$

$$\frac{F_K}{F_\pi} = 1 - \frac{1}{2}\mu_K - \frac{3}{4}\mu_\eta + \frac{5}{4}\mu_\pi + 4\,(m_s - \hat{m})\,\frac{B_0}{F_\pi^2}\,L_5^r$$

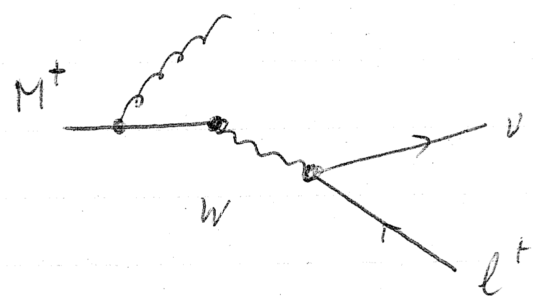
$$\mu_i = \frac{m_i^2}{32\pi^2 F_\pi^2}\log\frac{m_i^2}{\mu^2}$$

so $\dfrac{F_K}{F_\pi} \to L_5^r(\mu)$

(II) $\left.\begin{matrix}\pi_{\ell 2\gamma} \\ K_{\ell 2\gamma}\end{matrix}\right\}$ here there are 2 types of contributions:

(M meson is π or K)

$M(p) \to \ell^+(p_\ell)\,\nu_\ell(p_\nu)\,\gamma(q)$

a) 

$W = p_\ell + p_\nu$

Bremsstrahlung : must be there

b) Structure dependent part

$$\mathcal{M}_{SD}\quad T = +i\,G_F\,e\,V^*_{\substack{ud\\us}}\,\varepsilon^*_\mu\,H^{\mu\nu}\,\bar{\nu}\,\gamma_\nu(1+\gamma_5)\,e$$

$$H^{\mu\nu} = i\,V\,\varepsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta - A\left(q\cdot W\,g^{\mu\nu} - W^\mu q^\nu\right)$$

V can be related to $\pi^0 \to \gamma\gamma$
A can be related to $L_9 + L_{10}$ and in fact serves to determine it.
From $\pi^+ \to l\nu\gamma$ we can then predict $K^+ \to l\nu\gamma$

The γ can be off-shell, i.e. it is e^+e^- or $\mu^+\mu^-$
the extra piece can to $O(p^4)$ be fully expressed in terms of $F_V^K(q^2)$
So it is fully predicted.
Agreement with experiment is good
(but not with PDG values)
The expressions are rather long.

III

$$\begin{cases} \pi^+ \to \pi^0 e^+\nu \\ K^+ \to \pi^0 (e^+\nu) \quad (\mu^+\nu) \\ K^0 \to \pi^- (e^+\nu) \quad (\mu^+\nu) \end{cases}$$

These have the general structure

$$A = \frac{G_F}{\sqrt{2}} V_{us}^* \quad \bar{\nu}\gamma^\mu(1+\gamma_5)e \quad \langle \underset{p'}{\pi^{\,0}_{\,+}} | V_\mu | \underset{p}{K^{+}_{0}} \rangle$$

$$\frac{1}{\sqrt{2}}\left[(p'+p)_\mu f_+ + (p-p')_\mu f_- \right]$$

f_+ and f_- depend on $(p_e + p_\nu)^2$

and can be calculated using the CHPT formalism

PCAC gives $f_+ = 1$

$f_- = 0$

at next to leading order a good approximation is

$$f_+(t) = f_+(0)\left[1 + \lambda_+ \frac{t}{m_{\pi^+}^2}\right]$$

$$f_-(t) = f_-(0) = f_+(0)\,(\lambda_0 - \lambda_+)\,\frac{M_K^2 - M_\pi^2}{m_{\pi^+}^2}$$

with $\lambda_+ \simeq 0.031$ $(.030 \pm .002)$ $K^+ \to \pi^0 e^+ \nu$

$\lambda_0 \simeq 0.017$ $(.019 \pm .004)$ in $K^+ \to \pi^0 \mu^+ \nu$ decays

$f_-(t)$ only contributes $\propto m_\ell$ so is negligible in $K^+ \to \pi^0 e \nu$

there are small corrections from $f_+(0) = 1$ (vector symmetry)

IV $K_{\ell 3 \gamma}$: $K^{+}_{0} \to \pi^{0}_{-} \ell^{+} \nu \gamma$

lots of new form factors (10 in total)
all calculated + quite nontrivial results
good tests of chiral symmetry

V $K_{\ell 4}$ decays

$K \to \pi\pi\ e\nu,\ \mu\nu$
4 formfactors per decay : all known to next-to-leading order
+ some to higher orders