Nonleptonic Decays : Mixing

In some cases there is no difference in <u>conserved</u> quantum numbers between a particle and its antiparticle or between two different particles. In that case the 2 particles generally mix. This can lead to very interesting interference effects.
For the particle antiparticle mixing there are several candidates in the standard model:

$$\begin{cases} K^0 - \overline{K^0} & \bar{s}d \leftrightarrow s\bar{d} \\ D^0 - \overline{D^0} & c\bar{u} \leftrightarrow \bar{c}u \\ B^0 - \overline{B^0} & \bar{b}d \leftrightarrow b\bar{d} \\ B_s - \overline{B}_s & \bar{b}s \leftrightarrow b\bar{s} \end{cases}$$

I will describe the analysis for the K^0-$\overline{K^0}$ but its is identical for the others

$$\begin{cases} C|K^0\rangle = |\overline{K^0}\rangle \\ CP|K^0\rangle = -|\overline{K^0}\rangle \end{cases}$$

$$|K\rangle = A(t)\,|K^0\rangle + B(t)\,|\overline{K^0}\rangle \quad \text{and} \quad \psi(t) = \begin{pmatrix} A(t) \\ B(t) \end{pmatrix}$$

$$i\frac{d\psi}{dt} = \left(M - \frac{i}{2}\Gamma\right)\psi \quad \text{and} \quad M - \frac{i}{2}\Gamma = \frac{1}{2m_K}\langle K^0\ \overline{K^0}|\,H_{eff}\,|\begin{smallmatrix} K^0 \\ \overline{K^0} \end{smallmatrix}\rangle$$

Hermitian anti-Hermitian (remember Kaons decay)

$$M = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} \qquad \Gamma = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

we have used CPT to set the diagonal elements equal.

CP would require M_{12} and Γ_{12} to be real.
The diagonal elements are:

$$|K_{\substack{L\\S}}> = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0> \pm q|\bar{K}^0>\right) = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K^0_{\mp}> + \varepsilon|K^0_{\pm}>\right)$$

$$p^2 = M_{12} - \frac{i}{2}\Gamma_{12}$$

$$q^2 = M^*_{12} - \frac{i}{2}\Gamma^*_{12}$$

$$= \frac{1}{\sqrt{2}}\left(|K^0> \pm |\bar{K}^0>\right) \qquad \text{if CP invariance holds}$$

with $\varepsilon = \frac{p-q}{p+q} \simeq \frac{i}{2}\,\frac{\mathrm{Im}\,M_{12} - \frac{i}{2}\mathrm{Im}\,\Gamma_{12}}{\mathrm{Re}\,M_{12} - \frac{i}{2}\mathrm{Re}\,\Gamma_{12}}$

$$|K^0(t)> = g_+(t)\,|K^0> + \frac{q}{p}\,g_-(t)\,|\bar{K}^0>$$

$$|\bar{K}^0(t)> = \frac{p}{q}\,g_-(t)\,|K^0> + g_+(t)\,|\bar{K}^0>$$

$$g_\pm(t) = \frac{1}{2}\,e^{-\Gamma_L t/2}\,e^{-i m_L t}\underbrace{\left[1 \pm e^{-\Delta\Gamma\frac{t}{2}}\,e^{i\Delta m t}\right]}_{\text{this is what you would like to measure}}$$

$|K^0>$ identify by $\bar{s} \to \bar{u}\, l^+ \nu$

$|\bar{K}^0>$ $\qquad\qquad s \to u\, l^- \bar{\nu}$

charged lepton asymmetry, or any other decay mode.

This also shows how $\Delta\Gamma$ and Δm can be known accurately even if the width and the mass of the individual states is not known with the same precision.

at present :

K $\quad \Delta m \simeq (3.484 \pm .006) \times 10^{-12}\ \mathrm{MeV} \quad \simeq 2\,\mathrm{Re}\, M_{12}$

$\quad m \simeq 497.67 \pm .03\ \mathrm{MeV}$

$\quad \Delta\Gamma \simeq \Gamma_S \quad \simeq 2\,\mathrm{Re}\,\Gamma_{12}$

D $\quad |\Delta m| \cancel{\lesssim 1.4 \times 10^{-10}}\ \mathrm{MeV} = 9.[illegible] \pm 3. \times 10^{-12}\ \mathrm{MeV}$

$\quad m \simeq 1864.6 \pm 0.5\ \mathrm{MeV}$

$\quad \frac{\Delta\Gamma}{\Gamma} \lesssim \cancel{0.17}\ 0.0160 \pm 0.0025$

B $\quad : \frac{\Delta\Gamma}{\Gamma}$ is very small

$$\Rightarrow P_{total}(B \to \bar{B}_q) = \int_0^\infty |g_-(t)|^2\, dt = \frac{x_q^2}{2(1+x_q^2)} = \chi$$

$$x_q = \frac{\Delta M}{\Gamma}$$

$\chi_s = .62 \pm .13$ so x_s is large $(B_s \leftrightarrow \bar{B}_s)$

$\quad x_s > 2$ from time dependence directly

$\chi_b = .156 \pm .024$ or $x_d = 0.71 \pm 0.06$

$\Delta m_{B_0} = .51 \pm .06\ \ 10^{12}\ \hbar\, s^{-1}$

CPT : $\quad \left| \frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} \right| \leq 4 \times 10^{-18}$

$\Delta m_{B_0} = 3.337 \pm 0.033 \quad \times 10^{-10}\ \mathrm{MeV}$

$\Delta m_{B_s} = 1.164 \pm 0.005 \quad \times 10^{-8}\ \mathrm{MeV}$

Now at t_f it decays to state 1 : $B^0_f \to A_f$

$\overline{B^0_f} \to \overline{A_f}$

at t_b it decays to state 2 : $B^0_b \to A_f$

$\overline{B^0_b} \to \overline{A_f}$

$$A(t_f, t_b) = \frac{\sin\theta}{\sqrt{2}}\, e^{-\left(\frac{\Gamma}{2} + iM\right)\frac{t_f+t_b}{2}} \left\{ \cos\left[\frac{\Delta m_b}{2}(t_f - t_b)\right](A_f \overline{A_b} - \overline{A}_f A_b) - i\sin\left[\frac{\Delta m_B}{2}(t_f - t_b)\right]\left(\frac{p}{q}A_f A_b - \frac{q}{p}\overline{A_f}\,\overline{A_b}\right)\right\}$$

The funny sign in the Babar-book is for $t_1, t_2 = t_f, t_b$ $A_1 A_2 = A_f A_b$
or $t_2, t_1 = t_f, t_b$ $A_2 A_1 = A_f A_b$

which cancels out in the square anyway

Squaring this ; integrating over θ gives $\left(c = \cos\frac{\Delta m_B}{2}(t_f - t_b) \quad s = \sin\frac{\Delta m_B}{2}(t_f - t_b)\right)$

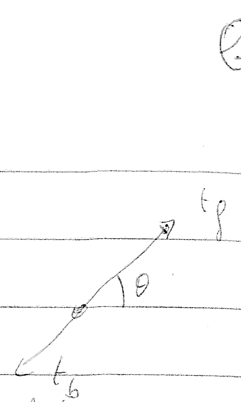
$$R(t_1, t_2) = C e^{-\Gamma(t_1+t_2)} \left[c^2 \left\{ |A_1|^2 |\overline{A}_2|^2 + |\overline{A}_1|^2 |A_2|^2 - 2\,\mathrm{Re}\, A_1 \overline{A}_2 \overline{A}_1^* A_2^* \right\} + s^2 \left\{ |A_1|^2 |A_2|^2 + |\overline{A}_1|^2 |\overline{A}_2|^2 - 2\,\mathrm{Re}\,\frac{pq^*}{qp^*} A_1 A_2 \overline{A}_1^* \overline{A}_2^* \right\} + 2cs\,\mathrm{Im}\left\{ (A_1\overline{A}_2 - \overline{A}_1 A_2)\left(\frac{p^*}{q^*} A_1^* A_2^* - \frac{q^*}{p^*}\overline{A}_1^* \overline{A}_2^*\right)\right\}\right]$$

We now use $\cos(2\alpha) = 1 - 2\sin^2(\alpha) = 2\cos^2\alpha - 1$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$ to rewrite this in the form of (1.40)

$\mathrm{Im}\,\frac{p^*}{q^*} = -\mathrm{Im}\,\frac{p}{q} = \mathrm{Im}\,\frac{q}{p}$ (we had $\left|\frac{q}{p}\right| = 1$)

In the Υ rest frame:



$$A(t_g, t_b) = \frac{\sin\theta}{\sqrt{2}} \left\{ B^0(t_g)\, \overline{B^0}(t_b) - \overline{B^0}(t_g)\, B^0(t_b) \right\}$$

the $\sin\theta$ factor is simply from the random distribution in θ

$$B^0(t) = g_+(t)\, B^0 + \frac{q}{p}\, g_-(t)\, \overline{B^0}$$

$$\overline{B^0}(t) = \frac{p}{q}\, g_-(t)\, B^0 + g_+(t)\, \overline{B^0}$$

we keep the label g, b to identify which one goes in the $0 < \theta < \frac{\pi}{2}$ part.

$$g_+ = e^{-iMt}\, e^{-\Gamma t/2} \cos\left(\frac{\Delta m_B\, t}{2}\right) \equiv c(t)$$

$$g_- = e^{-iMt}\, e^{-\Gamma t/2}\, i \sin\left(\frac{\Delta m_B\, t}{2}\right) = i\, s(t)$$

$$A(t_g, t_b) = \frac{\sin\theta}{\sqrt{2}}\, e^{-iM(t_g + t_b)}\, e^{-\Gamma \frac{t_g + t_b}{2}}$$

$$\begin{aligned}
\Big(& B^0_g\, \overline{B^0_b}\, \big(c(t_g)\, c(t_b) + s(t_g)\, s(t_b) \big) \\
& \overline{B^0_g}\, B^0_b\, \big(- s(t_g)\, s(t_b) - c(t_g)\, c(t_b) \big) \\
& B^0_g\, B^0_b\, \big(\tfrac{p}{q}\, i\, c(t_g)\, s(t_b) - \tfrac{p}{q}\, i\, s(t_g)\, c(t_b) \big) \\
& \overline{B^0_g}\, \overline{B^0_b}\, \big(\tfrac{q}{p}\, i\, s(t_g)\, c(t_b) - \tfrac{q}{p}\, i\, c(t_g)\, s(t_b) \big) \Big)
\end{aligned}$$

$$= \frac{\sin\theta}{\sqrt{2}}\, e^{-iM(t_g + t_b)}\, e^{-\frac{\Gamma}{2}(t_g + t_b)}$$

$$\times \Big[\cos \frac{\Delta m_B}{2}(t_g - t_b) \left[B^0_g\, \overline{B^0_b} - \overline{B^0_g}\, B^0_b \right]$$

$$- i \sin \frac{\Delta m_B}{2}(t_g - t_b) \left[\frac{p}{q}\, B^0_g\, B^0_b - \frac{q}{p}\, \overline{B^0_g}\, \overline{B^0_b} \right] \Big]$$