CP violation : Direct + in mixing

$\varepsilon \neq 0$: take K_L (just wait long enough to get rid of all K_S)

$$\frac{\Gamma(K_L \to \pi^- e^+ \nu_e)}{\Gamma(K_L \to \pi^+ e^- \nu_e)} \simeq \frac{1+2\,\mathrm{Re}\,\varepsilon}{1-2\,\mathrm{Re}\,\varepsilon} \simeq 1+4\,\mathrm{Re}\,\varepsilon$$

this gives $\mathrm{Re}\,\varepsilon = (1.635 \pm 0.060) \times 10^{-3}$

~~in~~direct :

$$A(K^0 \to (\pi\pi)_{I=0,2}) = A_{0,2}\, e^{i\delta_{0,2}}$$

$$A(\bar{K}^0 \to (\pi\pi)_{I=0,2}) = A^*_{0,2}\, e^{i\delta_{0,2}}$$

these $\pi\pi$ are $CP=+$ final states
we choose K^0 phase to have A_0 real.

then $$\eta_{+-} = \frac{A(K^0_L \to \pi^+\pi^-)}{A(K^0_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'$$

with $\varepsilon' = i\,\dfrac{\mathrm{Im}\,A_2\, e^{i(\delta_2-\delta_0)}}{\sqrt{2}\,A_0}$

we used $|\varepsilon'| \ll |\varepsilon|$
so from $|\eta_{+-}|$ and $|\eta_{00}|$ ~~one~~ we can get

$|\varepsilon|$ and $\mathrm{Re}\,\dfrac{\varepsilon'}{\varepsilon}$

$$|\varepsilon| = (2.263 \pm 0.023) \times 10^{-3}$$

$$\left|\frac{\varepsilon'}{\varepsilon}\right| = \begin{array}{ll}(14.7 \pm 2.2) \times 10^{-4} & (\text{CERN}) \\ (20.7 \pm 2.8) \times 10^{-4} & (\text{Fermilab})\end{array}$$ (using information on the phase)

Both types of measurements can be combined by producing a clean $K^0\bar{K}^0$ state at $t=0$ and look at the time dependence of all these effects, see the DAΦNE report.

As an example : $B_0, \bar{B}_0 \to \psi K_S$

$$A(B^0 \to f) = A \qquad f : \psi K_S$$
$$A(\bar{B}^0 \to f) = \bar{A}$$

has $|A| = |\bar{A}|$ (CPT)

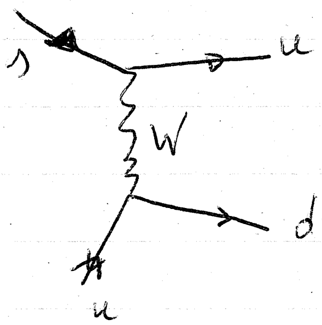
$$A_f = \int_0^\infty dt \, \frac{B_0(t) \to f \; - \; \bar{B}_0(t) \to f}{B_0(t) \to f \; + \; \bar{B}_0(t) \to f} = -\frac{x}{1+x^2} \sin \varphi_0 = -\frac{x}{1+x^2} \frac{\eta}{(1-\rho)^2 + \eta^2}$$

where $V_{td} V_{tb}^* V_{cs}^* V_{cb} = |V_{td} V_{tb}^* V_{cs}^* V_{cb}| \, e^{-i\varphi_0}$

This type of time dependent measurements is the main physics reason behind DAΦNE and the B-factories

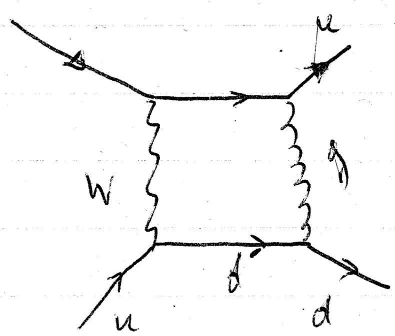
Nonleptonic decays: Short-distance

let us look first perturbatively



$$\rightarrow \quad O_A = \bar{d}\,\gamma_\mu (1+\gamma_5)\, u \;\; \bar{u}\, \gamma^\mu (1+\gamma_5)\, s$$

add



$$\rightarrow \quad - \frac{g_3^2}{16\pi^2} \ln\left(\frac{M_W^2}{\mu^2}\right) \left(3 O_B - O_A \right)$$

with $O_B = \bar{d}\,\gamma_\mu (1+\gamma_5)\, s \;\; \bar{u}\, \gamma^\mu (1+\gamma_5)\, u$

so gluonic corrections produce new operators

$$O_\pm \longrightarrow c_\pm\, O_\pm$$

with $$O_\pm = \frac{1}{2}\left(O_A \pm O_B\right)$$

$$c_\pm = 1 + d_\pm \frac{g_3^2}{16\pi^2} \log \frac{M_W^2}{\mu^2}$$

$d_+ = -2$ and $d_- = 4$

notice for $\mu \simeq 1$ GeV:

$$\left.\begin{array}{l} c_- \rightarrow 2.1 \\ c_+ \rightarrow 0.4 \end{array}\right\} \text{short distance perturbative enhancements}$$

O_+ is $\Delta I = \frac{3}{2}$, O_- is $\Delta I = \frac{1}{2}$ (exercise : check)

Now the change is very large because $\log \frac{M_W^2}{\mu^2}$ is so large

so we need to resum these contributions.
We do this using the renormalization group :

a general Green's function (bare!)

$$\mu_R \frac{d}{d\mu_R} \langle 0 | T (O_n \; \bar{q}_1 q_2 \bar{q}_3 q_4) | 0 \rangle_{unren} = 0$$

or using the μ dependence of the q fields: wave function renormalization

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_{QCD} \frac{\partial}{\partial g_{3r}} - 4 \gamma_F \right) \langle 0 | T (O_n \bar{q}_1 q_2 \bar{q}_3 q_4) | 0 \rangle_{ren} = 0$$

if in addition $O_n^{ren} = Z_k \, O_n^{unren}$ there is an additional term with γ_k if $Z_k = 1 + \gamma_k \ln \mu_R$ such that

$$O_n^{ren} (\mu) = c (\mu, \mu') \; O_n^{ren} (\mu')$$

satisfies

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_{QCD} \frac{\partial}{\partial g_{3r}} - 4 \gamma_F + \gamma_k \right) c (\mu, \mu') = 0 \qquad (1)$$

we know $c \; (\mu , \mu') = 0$
from our previous analysis $\gamma_{k\pm} = d_\pm \frac{g_3^2}{16 \pi^2} + 4 \gamma_F$

or $c_\pm (\mu \simeq 1 \text{ GeV}) = \begin{matrix} 1.5 \\ 0.8 \end{matrix}$

$$\beta_{QCD} = -\frac{g_3^2}{16\pi^2}\, b + \dots$$

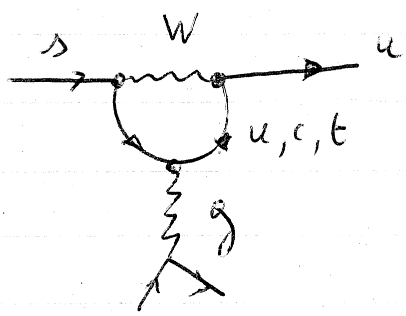
(1) is solved by

$$\frac{c_\pm(\mu)}{c_\pm(\mu')} = \left(1 + \frac{g_3^2}{16\pi^2}\log\frac{\mu'^2}{\mu^2}\right)^{d_\pm/b}$$

In fact life is (like usual) even more complicated:

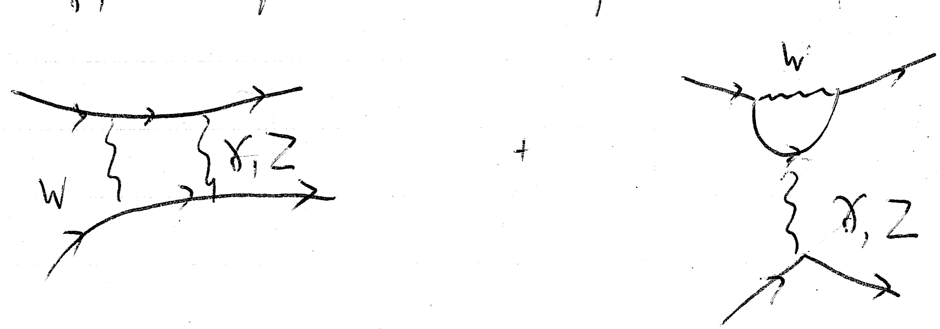
a) $b = 11 - \frac{2}{3} n_f$ changes when we cross thresholds

b) Penguin contributions: these generate additional operators via



- these are pure $\Delta I = 1/2$
- all generations now appear in the loop: the CP violating phase can occur.

c) γ, Z loops can be important too:



notice that the Penguin diagrams also bring in top dependence.

These anomalous dimensions are now known to 2 loop

A. Buras et al., NPB 400(93) 37

400(93) 75

370(92) 69

408(93) 209

+ confirmed by Martinelli et al., PLB 301(93) 263

+ references.

Penguins give rise to $\varepsilon' \neq 0$ since they can be CP violating in the full case.

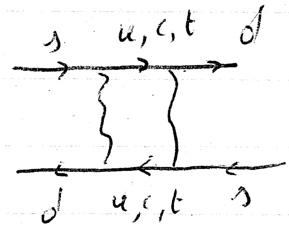
$$\frac{\varepsilon'}{\varepsilon} \sim \underset{\substack{\uparrow \\ m_t \gtrsim 200\,\text{GeV}}}{0} \sim 30 \times 10^{-4}$$

$\hookrightarrow m_t$ light

with lots of cancellations at present.

$\Delta S = 2$ Short distance:

At scale $\mu \simeq m_W^2$



$\mu \neq m_W^2$ we can run in a similar fashion as for $\Delta S = 1$

$$H^{box}_{W\,eff} = \frac{G_F^2}{16\pi^2}\Big[\left(V^*_{cd} V_{cs}\right)^2 m_c^2 \, \eta_1 \, H(x_c) \qquad \left(\text{notice } \frac{1}{M_W^2}\right)$$
$$+ 2\left(V^*_{td} V_{ts} V^*_{cd} V_{cs}\right) m_c^2 \, \eta_3 \, G$$
$$+ \left(V^*_{td} V_{ts}\right)^2 H(x_t) \, m_t^2 \, \eta_2 \Big] \, O_{\Delta S=2}$$

$$x_i = \frac{m_i^2}{m_W^2} \qquad \eta_1 \simeq .85$$

$$\eta_2 \simeq .62$$
$$\eta_3 \simeq .36$$

we have used here the GIM cancellation that follows if $m_c = m_u = m_t$ to remove the up contributions.

Imaginary part is in η_2, η_3 term $\Rightarrow$ ε

Real part is η_1 term $\Rightarrow$ ΔM

if we can get $\langle K^0 | \, O_{\Delta S=2}(\mu) \, | \bar{K}^0 \rangle \equiv \frac{16}{3} F_K^2 m_K^2 B(\mu)$

with $O_{\Delta S=2} = \bar{d}\gamma_\mu(1+\gamma_5)s \; \bar{d}\gamma^\mu(1+\gamma_5)s$

Similar analysis can be made for $b \to c, u$ and $c \to s, d$ decays and $D_0 - \bar{D}_0$ mixing and $B_0 - \bar{B}_0$ mixing.

$D_0 - \bar{D}_0$ is expected to be small: m_d, m_s, m_b are small ($\ll m_W^2$) compared to $B_0 - \bar{B}_0$, $K_0 - \bar{K}_0$ mixing

There are also $b \to s$ $\gamma + g$ transitions via the Penguin Diagrams.

These have in fact been observed by CLEO at Cornell.

$b \to s\gamma$ is $2\text{-}4 \times 10^{-4}$ of the quark decays. (*)

$B \to K^* \gamma$ is observed at $4.5 \pm 1.8 \times 10^{-5}$

$b \to s\gamma$ leads to a peak of γ's at roughly $\frac{m_B}{2}$

this has also been used to extract a branching ratio in agreements with (*)

notice that this puts a limit on $|V_{ts} / V_{cb}| = 1.1 \pm .6$ in agreement with unitarity.

Some attempts have been made to search for $b \to s g$

Some remarks about long distance aspects.

So previously we have seen that the short distance evolution gives rise to an effective Lagrangian (or Hamiltonian) of the type

$$H_W = \sum_i c_i(\mu) O_i(\mu)$$

To obtain decays we then have to calculate the matrixelements

eg: $$\langle \pi^+\pi^0 | H_W | K^+ \rangle = \sum_i c_i(\mu) \langle \pi^+\pi^0 | O_i(\mu) | K^+ \rangle$$

so the μ-dependence of the matrixelements should be such that the $c_i(\mu)$ dependence gets cancelled.
This is precisely where most models fail since they do not have an obvious μ-dependence.
There are various attempts (including by myself) to introduce this μ-dependence in models but at present none is very satisfactory.

One simple approximation is the vacuum saturation approach:

$$\langle K_0 | \bar{d}^\alpha \gamma_\mu (1+\gamma_5) s^\alpha \bar{d}^\beta \gamma^\mu (1+\gamma_5) s^\beta | \bar{K}_0 \rangle$$

$$= \langle K_0 | \bar{d}^\alpha \gamma_\mu (1+\gamma_5) s^\alpha | 0 \rangle \langle 0 | \bar{d}^\beta \gamma^\mu (1+\gamma_5) s^\beta | \bar{K}^0 \rangle$$

$$+ \;\;〃\;\; \bar{d}^\alpha \;\;〃\;\; s^\beta \;\;〃\;\; \bar{d}^\beta \;\;〃\;\; s^\alpha$$

$$= \frac{16}{3} F_K^2 m_K^2$$

One approach is again just using the symmetry properties of H_W and construct an effective Lagrangian using U of CHPT with the same symmetry transformation properties.

This can in fact be done and is the reason why em Penguin effects can be as large as gluonic ones for CP violation in Kaon decays.

HQET can also be used to calculate relations between

$$B \to D\pi \quad \text{and} \quad B \to D^*\pi$$

in some cases it can be proven that factorization is actually correct, for the decay

$\overline{B^0} \to D^{*+}\pi^-$ this is the case (Dugan-Grinstein)

Chiral Perturbation Theory and Nonleptonic Decays

We saw that at the quark level (gluons only) there were 6 possible terms at the leading level in $1/M_W^2$ but to all orders in $\left[\log\left(\frac{M_W^2}{\mu^2}\right) \times \alpha_S(\mu^2)\right]$ (the latter using the renormalization group.)

We had 6 possible terms (after integrating out b, c, t quarks)

$$O_A = \bar{u}^\alpha \gamma_\mu (1-\gamma_5) s^\alpha \; \bar{d}^\beta \gamma^\mu (1-\gamma_5) u^\beta$$

$$O_B = \bar{d}^\alpha \gamma_\mu (1-\gamma_5) s^\alpha \; \bar{u}^\beta \gamma^\mu (1-\gamma_5) u^\beta$$

$$O_3 = \bar{d}^\alpha \gamma_\mu (1-\gamma_5) s^\beta \left[\bar{u}^\gamma \gamma^\mu (1-\gamma_5) u^\delta + \bar{d}^\gamma \gamma^\mu (1-\gamma_5) d^\delta + \bar{s}^\gamma \gamma^\mu (1-\gamma_5) s^\delta \right]$$

$$\times\, \delta^{\alpha\beta} \times \delta^{\gamma\delta}$$

$$O_4 = \ldots \qquad \times\, \delta^{\alpha\delta} \delta^{\beta\gamma} \qquad \text{(same as in } O_3\text{)}$$

O_5 = as O_3 but $1+\gamma_5$ in the 2nd side ($\bar{u}..u + \bar{d}..d + \bar{s}..s$)
O_6 = " O_4 " " "

$O_A - O_B$, O_3, O_4, O_5, O_6 belong to $(8_L, 1_R)$ of $SU(3)_L \times SU(3)_R$
$O_A + O_B$ has $(8_L, 1_R)$ & $(27_L, 1_R)$ parts

$$O_{\Delta S=2} = \bar{d}^\alpha \gamma_\mu (1-\gamma_5) s^\alpha \; \bar{d}^\beta \gamma_\mu (1-\gamma_5) s^\beta$$

is $(27_L, 1_R)$

Can we now construct a chiral Lagrangian that is $SU(3)_L \times SU(3)_R$ invariant using these operators?

We proceed here in a way similar to what we did for introducing the quark masses:

in $\mathcal{L}_{QCD}$ we had $\quad - m_{qi}\, \bar{q}^i q^i$

which we described in a chiral invariant way by introducing s, p external fields instead

$$- m_{qi}\, \bar{q}_i q_i \quad \longrightarrow \quad - \bar{q}_{Ri} (s + i p)_{ij}\, q_{Lj}$$

and $s + ip \to g_R (s + ip) g_L^+$ under an $SU(3)_L \times SU(3)_R$ transformation.

In a similar way we can introduce a t^{8ij} that transforms as an $(8_L, 1_R)$ (i.e.) $\quad t^8 \to g_L t^8 g_L^+$ and $\operatorname{tr} t^8 = 0$

and a $t^{27\, ii'jj'}$ $\quad t^{27\, ii'jj'} \longrightarrow g_L^{ki}\, g_L^{lj}\, (g_L^+)^{i'k'} (g_L^+)^{j'l'}\, t^{27\, kk'}_{\, ll'}$

with $\left.\begin{array}{l} t^{27\, ii'jj'} = t^{27\, ji'ij'} = t^{27\, ij'ji'} \\ \text{and } t^{27\, kkjj'} = 0 \end{array}\right\}$ makes it a 27_L

Note: to prove the various properties of $O_A, O_B, O_1, O_2, O_3, O_4$ for O_A and O_B one needs to use Fierz transformations.

Now we treated $s + ip$ in the CHPT Lagrangian as $\chi = 2 B_0 (s + ip)$ so a free constant in front of all the terms and only the symmetry is important.

We can now do the same for the weak decays
(first done by Cronin)
modern version (including p^4) Kambor, Missimer, Wyler, 91-92
and many more papers.

$$- c_2 \; t^{8ij} \left(U^\dagger D_\mu U U^\dagger D^\mu U\right)^{ji}$$

$$= c_2 \; \mathrm{tr}\left(t^8 D_\mu U D^\mu U^\dagger\right)$$

$$- c_3 \; t^{27\, ii' jj'} \left(U^\dagger D_\mu U\right)^{ii'} \left(U^\dagger D^\mu U\right)^{jj'}$$

- there is no term without derivatives $\Rightarrow$ a suppression in the matrix elements corresponding to m_K^2/Λ^2 appears

- there is a "weak mass" term as well

$$c_5 \; \mathrm{tr}\left(t^8 \left(\chi^\dagger U + U \chi^\dagger\right)\right)$$

this term does not contribute to K-decays (it is also called the tadpole contribution)

- for γ & Z exchange in the Penguins, there are possible $(8_L, 8_R)$ operators.
These allow terms like
$$\mathrm{tr}\; t_L^8 \, U^\dagger \, t_R^8 \, U$$
which are order p^0 hence the enhancement alluded to earlier.

- relating $\Delta S = 2$ & $\Delta S = 1$ can be done, we determine c_3 from $K \to \pi\pi(\pi)$ and then get B_K

this is unfortunately known to have large higher order corrections.

- At order p^4 there are a lot more things that can contribute but useful predictions remain.

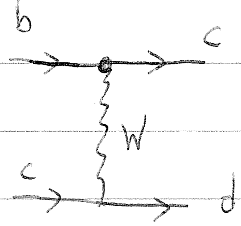
- It is also possible to combine chiral & heavy quark symmetries
- We can add baryons (p, n, hyperons) to the game

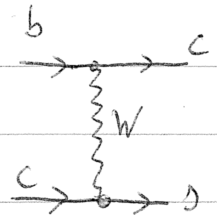
$\Rightarrow$ lots of places where this type of constraints is useful.

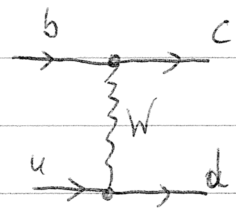
B-decays

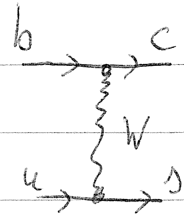
Well we now have several different effective Hamiltonians:

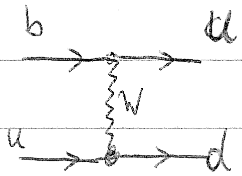
$\Delta B = +1$	$\Delta B = +1$	$\Delta B = +1$	$\Delta B = +1$	$\Delta B = +1$	$\Delta B = +1$
$\Delta S = 0$	$\Delta S = -1$	$\Delta S = 0$	$\Delta S = 0$	$\Delta S = -1$	$\Delta S = -1$
$\Delta C = 0$	$\Delta C = 0$	$\Delta C = +1$	$\Delta C = -1$	$\Delta C = +1$	$\Delta C = -1$
$\Delta U = 0$	$\Delta U = 0$	$\Delta U = -1$	$\Delta U = +1$	$\Delta U = -1$	$\Delta U = +1$

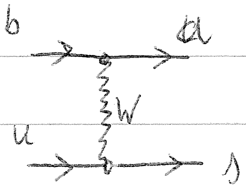


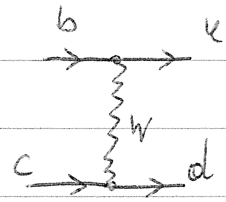


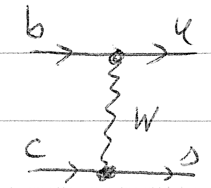












The first two have also Penguin Contributions, the last 4 not.

Exercise:
- check the possible Penguin diagrams
- why are there no Penguin's in the latter case
- can you think of diagrams that would produce CP violation for the latter.
  If yes; why are they universally neglected
- Write down the equivalent for the 10 operators of K-decay

(p28 Babar book)

In fact we have ~~one~~ two more operator now: the electro magnetic chromomagnetic Penguin

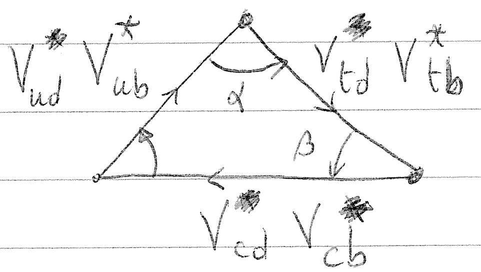
$$Q_{11} = m_B \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R \frac{e}{8\pi^2} \qquad Q_{12} = m_B \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R \frac{g}{8\pi^2}$$

typical numbers are $(\alpha(M_Z) = 0.118)$

	$\mu = 4.8$ GeV	9.6 GeV	$\mu = 4.8$ lowest order
C_1	-0.174	-0.100	-0.249
C_2	1.073	1.039	1.108
C_3	0.013	0.008	0.011
C_4	-0.034	-0.024	-0.026
C_5	0.009	0.006	0.008
C_6	-0.038	-0.0025	-0.031
C_{11}			-0.149

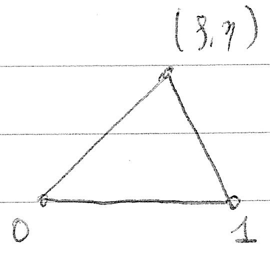
	LO	NDR
C_7/α	$+0.042$	-0.003
C_8/α	0.041	0.047
C_9/α	-1.264	-1.279
C_{10}/α	0.291	0.234
C_{11}	-0.299	
C_{12}	-0.143	

Now before we classify the decays let's remember the unitarity triangle



$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

or



whenever divided by $-A\lambda^3$

So ~~CP~~ will be large if the CKM angles in the real and imaginary part are roughly the same.

In the K-system: decay amplitude $\sim \lambda$

~~CP~~ " $\sim A^2 \lambda^6 \eta$

so ~~CP~~ is of order $\lambda^5 \sim 0.5 \times 10^{-3}$

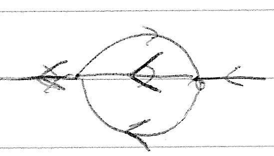
Inclusive Decays : Intermezzo

We can calculate some things on nonleptonic decays just as we did for the inclusive semileptonic decays.
So

$$\Gamma_{nl} = \mathrm{Im} \langle \bar{B}^0 | \, T(O_1 \, O_1^\dagger) \, | \bar{B}^0 \rangle$$

With here O_1 the full relevant effective Hamiltonian and this can be done for all decays.

calculable

$\langle \bar{B}^0 |$  $| \bar{B}^0 \rangle$

$$= \frac{3 G_F^2 m_b^5}{192 \pi^3} \Big\{ \langle \bar{B}^0 | \bar{b} b | \bar{B}^0 \rangle (\;) + \langle \bar{B}^0 | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} | \bar{B}^0 \rangle (\;) + \ldots \Big\}$$

This approach works fine but there are some problems (some gone)

1) $\left. \dfrac{\tau(\Lambda_b)}{\tau(B)} \right|_{HQE} \simeq 0.90 - 0.95$ and exp. 0.77 ± 0.04

2) charm counting expect # C per B decay to be 1.2 (2001)
measured 1.23 ± 0.04

3) $B_{sl} \simeq 11.5 - 12.5\,\%$ but measured $10.4\,\% \pm 0.29$
↓
can be lowered by choosing a small μ

$$\alpha = \arg\left(- \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right)$$

$$\beta = \arg\left(- \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

$$\gamma = \arg\left(- \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

and processes are classified via their various contributions:

tree

Penguin : QCD or EW

eg $\bar{b} \to c\bar{c}s = V_{cb} V_{cs}^* \left(T_{c\bar{c}s} + P_s^c - P_s^t \right) + V_{ub} V_{us}^* \left(P_s^u - P_s^t \right)$

where we used $V_{tb} V_{ts}^* = - V_{ub} V_{us}^* - V_u$

$$\begin{array}{ll}
b \to c\bar{c}s & V_{cb} V_{cs}^* \left(T_{c\bar{c}s} + P_s^c - P_s^t \right) + V_{ub} V_{us}^* \left(P_s^u - P_s^t \right) \\
b \to s\bar{s}s & V_{cb} V_{cs}^* \left(P_s^c - P_s^t \right) + V_{ub} V_{us}^* \left(P_s^u - P_s^t \right) \\
b \to u\bar{u}s & V_{cb} V_{cs}^* \left(P_s^c - P_s^t \right) + V_{ub} V_{us}^* \left(P_s^u - P_s^t + T_{u\bar{u}s} \right) \\
b \to d\bar{d}s & \text{(only via the Penguins} \\
b \to c\bar{u}s & V_{cb} V_{us}^* \quad T_{c\bar{u}s} \\
b \to u\bar{c}s & V_{ub} V_{cs}^* \quad T_{c\bar{u}s}
\end{array}$$

$$\begin{array}{ll}
b \to c\bar{c}d & V_{tb} V_{td}^* \left(P_d^t - P_d^u \right) + V_{cb} V_{cd}^* \left(T_{c\bar{c}d} + P_d^c - P_d^u \right) \\
s\bar{s}d & V_{tb} V_{td}^* \left(P_d^t - P_d^u \right) + V_{cb} V_{cd}^* \left(P_d^c - P_d^u \right) \\
u\bar{u}d & V_{tb} V_{td}^* \left(P_d^t - P_d^c \right) + V_{ub} V_{ud}^* \left(T_{u\bar{u}d} + P_d^u - P_d^c \right) \\
d\bar{d}d & \\
c\bar{u}d & V_{cb} V_{ud}^* \\
u\bar{c}d & V_{ub} V_{cd}^*
\end{array}$$

Example 1 : $B \to \psi K_S$ (a class 1 decay) : α

The underlying process is $b \to c\bar{c}s$: $\sim V_{cb} V_{cs}^*$

$\bar{b} \to c\bar{c}\bar{s}$ $\sim V_{cs} V_{cb}^*$

So $$\frac{\bar{A}}{A} = \underbrace{\eta_{\psi K_S}}_{\text{the CP of the final state}} \times \frac{V_{cb}}{V_{cs}} \frac{V_{cs}^*}{V_{cb}^*} \times \left(\frac{p}{q}\right)_K$$

The last factor is needed because the underlying decays are

$\bar{B}^0 \to J/\psi \bar{K}^0$ and $B^0 \to J/\psi K^0$

therefore we need an extra factor of $\left(\frac{p}{q}\right)_K$. Now all B_K factors etc. that show up in the ratio cancel.

We can therefore use $\left(\frac{p}{q}\right)_K = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \underbrace{e^{-2i\xi_K}}$ $\to$ the phase choice in $K^0 \leftrightarrow \bar{K}^0$

(note : it is the real part of the mixing that counts so the charm-quark part dominates)

So the asymmetry measure $|\lambda|$ and $\mathrm{Im}\,\lambda$

with $$\lambda = \left(\frac{p}{q}\right)_B \frac{\bar{A}}{A} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \left(\frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*}\right)^*$$

now looking at the definition of β we see $\lambda \approx e^{2i\beta}$

or $\mathrm{Im}\,\lambda = \sin 2\beta$

A class 2 example : β

Here $b \to u\bar{u}d$ comes into play via $B \to \pi^+\pi^-$

$$b \to u\bar{u}d = V_{tb}V_{td}^* \left(P_d^t - P_d^c\right) + V_{ub}V_{ud}^* \left(T_{u\bar{u}d} + P_d^u - P_d^c\right)$$

If only the tree level term is important (or rather the $V_{ub}V_{ud}^*$ term)

then
$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} e^{-2i\xi_B}$$
or when adding the B-$\bar{B}$ mixing to get to λ we have

$$\lambda = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*} = e^{2i\alpha}$$

The problem here is that $V_{ub}V_{ud}^* \approx A\lambda^3(\rho - i\eta)$

and the first term is $\underset{\substack{\uparrow \\ \text{expected size of Penguin.}}}{V_{tb}V_{td}^* \; 0.04} \approx A\lambda^3(1-\rho-i\eta)$

From $B \to K\pi$ the Penguin contribution seems to be enhanced. But measuring also $\overset{(-)}{B} \to \pi^0\pi^0$ and $B^\pm \to \pi^\pm\pi^0$ and using isospin the various contributions can in principle be disentangled.

In principle the $b \to d\bar{d}d$ Penguin diagrams also contribute somewhat complicating the analysis.

The isospin analysis is very similar to $\cancel{B}K \to \pi\pi$:

$$A(B^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2} A_2$$

$$A(B^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{6}} A_2 - \frac{1}{\sqrt{3}} A_0$$

$$A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} A_2 + \frac{1}{\sqrt{6}} A_0$$

So neglecting electroweak Penguins (which introduce different isospin structures) measuring 3 decays allows to disentangle the tree level contribution from the Penguin one. The tree level one ~~d~~ is the only one that contributes to A_2.

Notice that the 3 (complex) amplitudes obey a relation

$$A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00}$$

and similarly the CP conjugate ones

$$A^{-0} = \frac{1}{\sqrt{2}} \overline{A^{+-}} + \overline{A^{00}}$$

An example to measure γ

γ is in fact one of the more difficult quantities to measure.

I will look at a method using $B^\pm \to D K^\pm$ decays

The D^0-$\overline{D^0}$ system also has CP eigenstates

$$D^0_\pm = \frac{1}{\sqrt{2}}\left(D^0 \mp \overline{D^0}\right)$$

precisely as for $K^0 \overline{K^0}$ and $B^0 \overline{B^0}$. Remember that because of the GIM mechanism ~~CP~~ in D^0-$\overline{D^0}$ mixing via the box diagram is expected to be very small.

$$A(B^+ \to D^0 K^+) = V^*_{ub} V_{cs} \, |a| \, e^{i\Delta a} \qquad (\bar{b} \to \bar{u} c \bar{s})$$

$$A(B^- \to \overline{D^0} K^-) = V_{ub} V^*_{cs} \, |a| \, e^{i\Delta a} \qquad (b \to u \bar{c} s)$$

$$A(B^+ \to \overline{D^0} K^+) = V^*_{cb} V_{us} \, |A| \, e^{i\Delta A} \qquad (\bar{b} \to \bar{c} u \bar{s})$$

$$A(B^- \to D^0 K^-) = V_{cb} V^*_{us} \, |A| \, e^{i\Delta A} \qquad (b \to c \bar{u} s)$$

Remember now that in order to asymmetries we need 2 modes with different weak phases and different strong phases.

Now observing $\overset{(-)}{D}{}^0 \to \pi^+\pi^-, K^+K^-$ we know it's the D^0_- component which contributes.
That way we can can measure 6 modes

$B^+ \to D^0 K^+ \;;\; \bar{D}^0 K^+ \;;\; D^0_+ K^+$

$B^- \to D^0 K^- \;;\; \bar{D}^0 K^- \;;\; D^0_- K^-$

This allows to extract γ.

Exercise: show how

note: V_{us}, V_{ud} are known and real
V_{cb} is known and real to $O(\lambda^4)$ (prove)

Solution of exercise.

$|V_{ub}|\,|V_{cs}|\,|a|$

$|V_{cb}|\,|V_{us}|\,|A|$

$|a|\,|A|$ ~~arg~~ the $V_{ub}^* V_{cs} V_{cb} V_{us}^* e^{i(\Delta_a - \Delta_A)}$

and the $V_{ub} V_{cs}^* V_{cb}^* V_{us} e^{i(\Delta_a - \Delta_A)}$

allows to get angle of $\dfrac{V_{ub}^* V_{cs} V_{cb} V_{us}^*}{V_{ub} V_{cs}^* V_{cb}^* V_{us}}$

$\llcorner 1$ V_{us} is real.

now to a very good precision we can use the knowledge of absolute values of V_{us} (it's real in the usual parametrization) and V_{ub} (again real in the standard parametrization)

and V_{cs} : almost real : imaginary part starts at very high order like λ^4 and is known to be $1 - \frac{\lambda^2}{2}$ to good precision.

$\Rightarrow$ this allows to get λ

so one get $\left.\begin{array}{l}\cos(\gamma + \Delta_a - \Delta_A) \\ \text{and } \cos(-\gamma + \Delta_a - \Delta_A)\end{array}\right\} \Rightarrow$ gets γ with a fourfold ambiguity

V_{CD} real to $O(\lambda^4)$

$$\begin{pmatrix} X_1 & \lambda & A\lambda^3(\rho - i\eta) \\ X_2 & X_3 & A\lambda^2 \\ x & x & x \end{pmatrix}$$

now X_1 is real by phase definition $\Rightarrow$

$$\begin{cases} X_1 = 1 - \lambda^2 - A\lambda^6(\rho^2 + \eta^2) & \\ X_2^2 + X_3^2 + A\lambda^4 = 1 & (1) \\ X_2 X_1^* + X_3\lambda + A\lambda^5(\rho + i\eta) = 0 & (2) \end{cases}$$

X_2 cannot have $O(1)$ term from (2)
So X_3 must start at 1

$\Rightarrow$ X_2 starts out as $-\lambda$ with no imaginary part

.... but all phases should be determined by ρ, η
$\Rightarrow$ can only start at λ^4!