An example how matrices with different indices should be understood

One way to understand the presence of many different type of indices is to combine all of them into one superindex and thus write it as one very big matrix. We do not do this in practice since it requires a large amount of writing and obscures some of the reasoning. It can however be used to understand what's going on.

An example is $SU(2)_L$ indices and a generation index. i, j, k are $SU(2)_L$ indices and r, s, t generation indices. ρ, σ, τ denote the superindex.

i, j, k can take the value 1 or 2 (doublet).

r, s, t can take values 1, 2 or 3 (three generations).

ρ, σ can take the values 1,2,. . . ,6. (the six possible combinations of an i and an r index).

The matrix $W_\mu = W_\mu^a \tau^a = \begin{pmatrix} W_\mu^0 & -\sqrt{2}W_\mu^+ \\ -\sqrt{2}W_\mu^- & -W_\mu^0 \end{pmatrix}$ has i, j indices and it acts on the three generations.

Let me also introduce $V_{CKM} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & \\ 0 & 0 & 1 \end{pmatrix}$ as an example of a matrix with r, s, t indices. But it only acts on component with $SU(2)_L$ indices $i = 2$.

Then $\sum \overline{Q} W_\mu V_{CKM} Q$ really means $\overline{\tilde{Q}} \widetilde{W}_\mu \widetilde{V}_{CKM} \tilde{Q}$ with the $\tilde{A}$ quantities in the r, s, t indices with the correspondance

ρ	r	i
1	1	1
2	1	2
3	2	1
4	2	2
5	3	1
6	3	2

and

$$\overline{\tilde{Q}} = \begin{pmatrix} \overline{u} & \overline{d} & \overline{c} & \overline{s} & \overline{t} & \overline{b} \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} u \\ d \\ c \\ s \\ t \\ b \end{pmatrix}$$

$$\widetilde{W}_\mu = \begin{pmatrix} W_\mu^0 & -\sqrt{2}W_\mu^+ & 0 & 0 & 0 & 0 \\ -\sqrt{2}W_\mu^- & -W_\mu^0 & 0 & 0 & 0 & 0 \\ 0 & 0 & W_\mu^0 & -\sqrt{2}W_\mu^+ & 0 & 0 \\ 0 & 0 & -\sqrt{2}W_\mu^- & -W_\mu^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & W_\mu^0 & -\sqrt{2}W_\mu^+ \\ 0 & 0 & 0 & 0 & -\sqrt{2}W_\mu^- & -W_\mu^0 \end{pmatrix}$$

$$\widetilde{V}_{CKM} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$