Energy Conservation and Saturation in small-x Evolution

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BFKL Evolution

High-energy limit of hadronic scattering in QCD described by BFKL evolution.

Resums terms going as $(\alpha_s \log x)^n$ in a perturbative expansion.

Results in cross sections rising with increasing energy, or equivalently decreasing x.

Rise of the form $x^{-\lambda}$. To leading order λ given by $\alpha_s 4 \log 2 \sim 0.5$ for $\bar{\alpha} \equiv 3\alpha_s/\pi = 0.2$.

BFKL Evolution

The fast rise of σ violate unitarity bounds at low x.

Non-linear effects from gluon recombinations dampens the growth and cause the gluon density to saturate.

NLO corrections to BFKL turn out to be very large. These corrections strongly suppress the growth of σ for small-x

Large fraction of these NLO corrections related to energy conservation.

Understanding LHC Physics

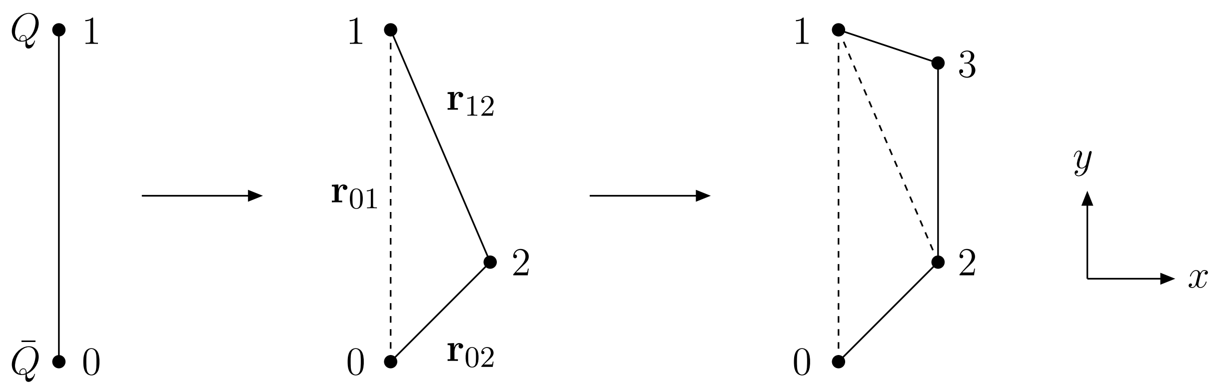
Important in the understanding of high density QCD at LHC range of energies.

Is the reduction in the growth rate due to saturation or energy conservation?

When do saturation effects become important?

Also essential in understanding high energy cosmic rays.

Mueller's Dipole Model



$$\gamma^* \to Q\bar{Q} \to Qg\bar{Q} \to Qgg\bar{Q} \to \dots$$

Formulated in transverse coordinates, $\mathbf{r}$, and reproduces LL BFKL evolution.

Also easy to take into account unitarisation effects due to multiple pomeron exchange.

Mueller's Dipole Model

Probability for a dipole splitting given by

$$\frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi} d^2\mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2\, r_{12}^2} \cdot S$$

$$\text{where } S = \exp\left[-\frac{\bar{\alpha}}{2\pi}\int dy \int d^2\mathbf{r}_2 \frac{r_{01}^2}{r_{02}^2\, r_{12}^2}\right].$$

Singular for small r_{02} and r_{12}.

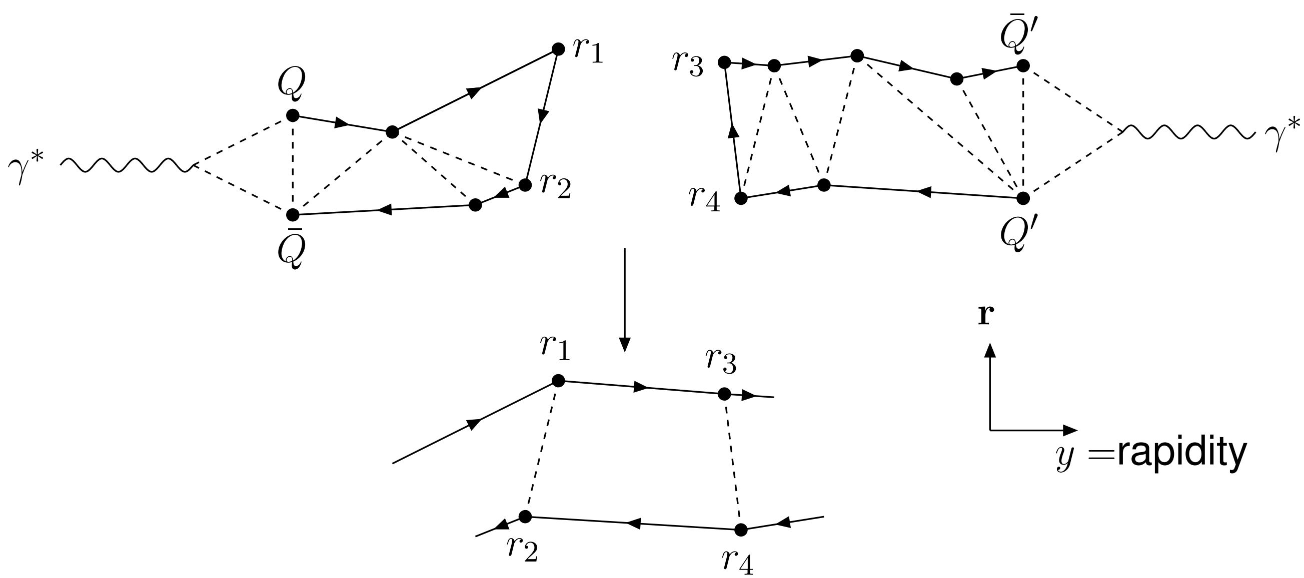
Mueller introduced a cut-off, ρ, such that $r_{02} > \rho$ and $r_{12} > \rho$.

Small ρ implies many small dipoles but these interact very weakly so that total σ finite even when $\rho \to 0$.

Dipole Interactions

The weight for a dipole-dipole interaction is given by

$$f = \frac{\alpha_s^2}{2}\left\{\log\left[\frac{|\mathbf{r}_1 - \mathbf{r}_3|\cdot|\mathbf{r}_2 - \mathbf{r}_4|}{|\mathbf{r}_1 - \mathbf{r}_4|\cdot|\mathbf{r}_2 - \mathbf{r}_3|}\right]\right\}^2.$$



Cross Section

Unitarity easily implemented by the replacement $f \to 1 - e^{-f}$.

Total cross section then given by

$$\sigma \sim \int d^2\mathbf{b}(1 - e^{-\sum f_{ij}}).$$

The Linked Dipole Chain Model

A model for DIS which interpolates between BFKL and DGLAP evolutions.

Main feature is the observation that both the total σ and the structure of the final state is decided by a set of carefully selected initial gluons.

These are ordered in both p_+ and p_-.

Remaining final state gluons can be treated as FSR from these primary gluons. The FSR do not modify the total σ.

Relation Between LDC and Mueller

In Mueller's cascade:

Identify transverse momentum with the inverse dipole size $1/r$.

Require the same ordering of the primary gluons as in the LDC model.

$\Longrightarrow$ The same weight as in LDC for the emission probabilities are obtained.

Conserving Energy-Momentum

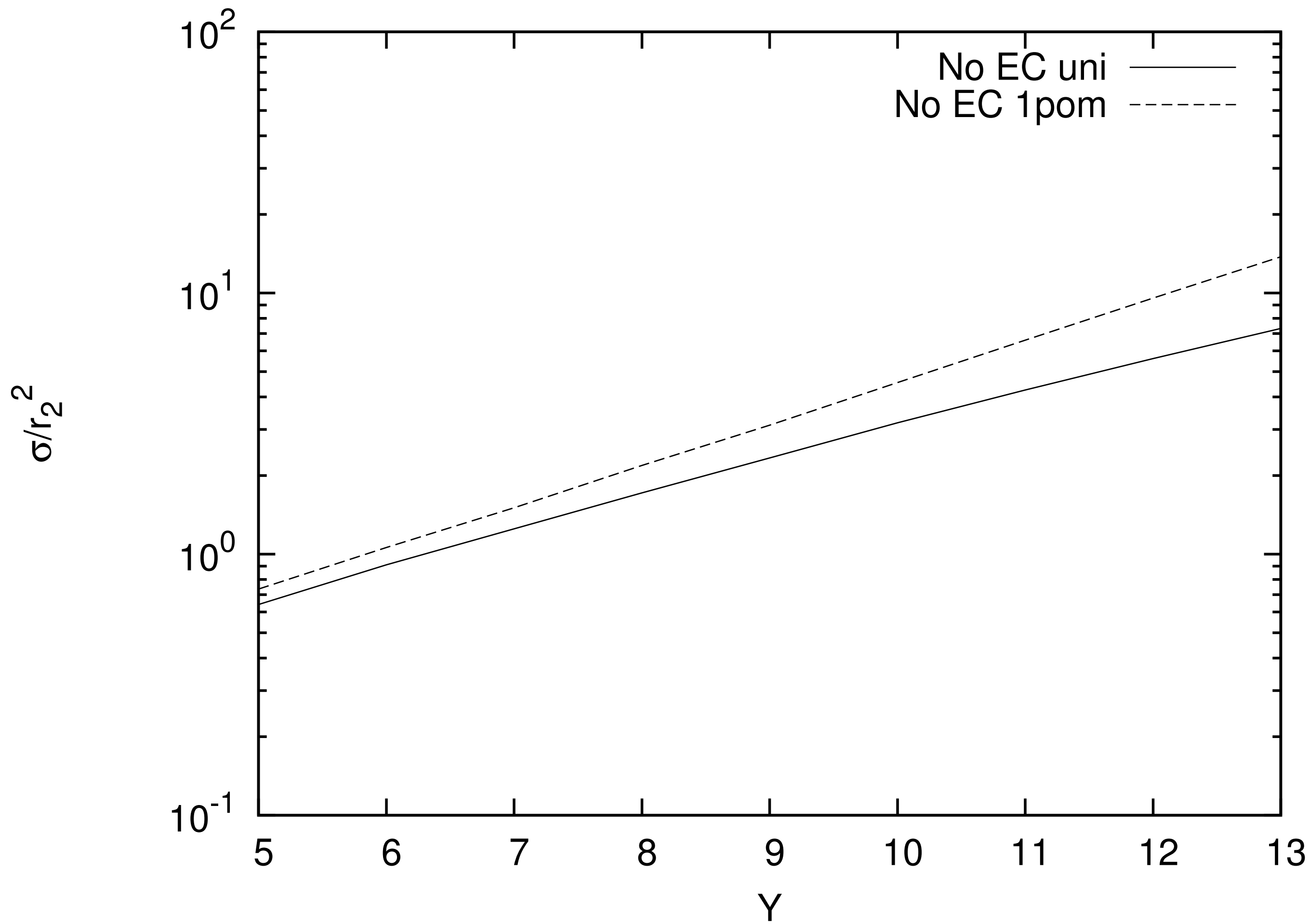
$\Longrightarrow$ Identifying a subset of gluons in Mueller's cascade can correspond to the primary gluons in momentum space cascade.

These determine σ and rest of the emissions can be regarded as virtual fluctuations or FSR.

It is necessary that these primary gluons conserve energy-momentum.

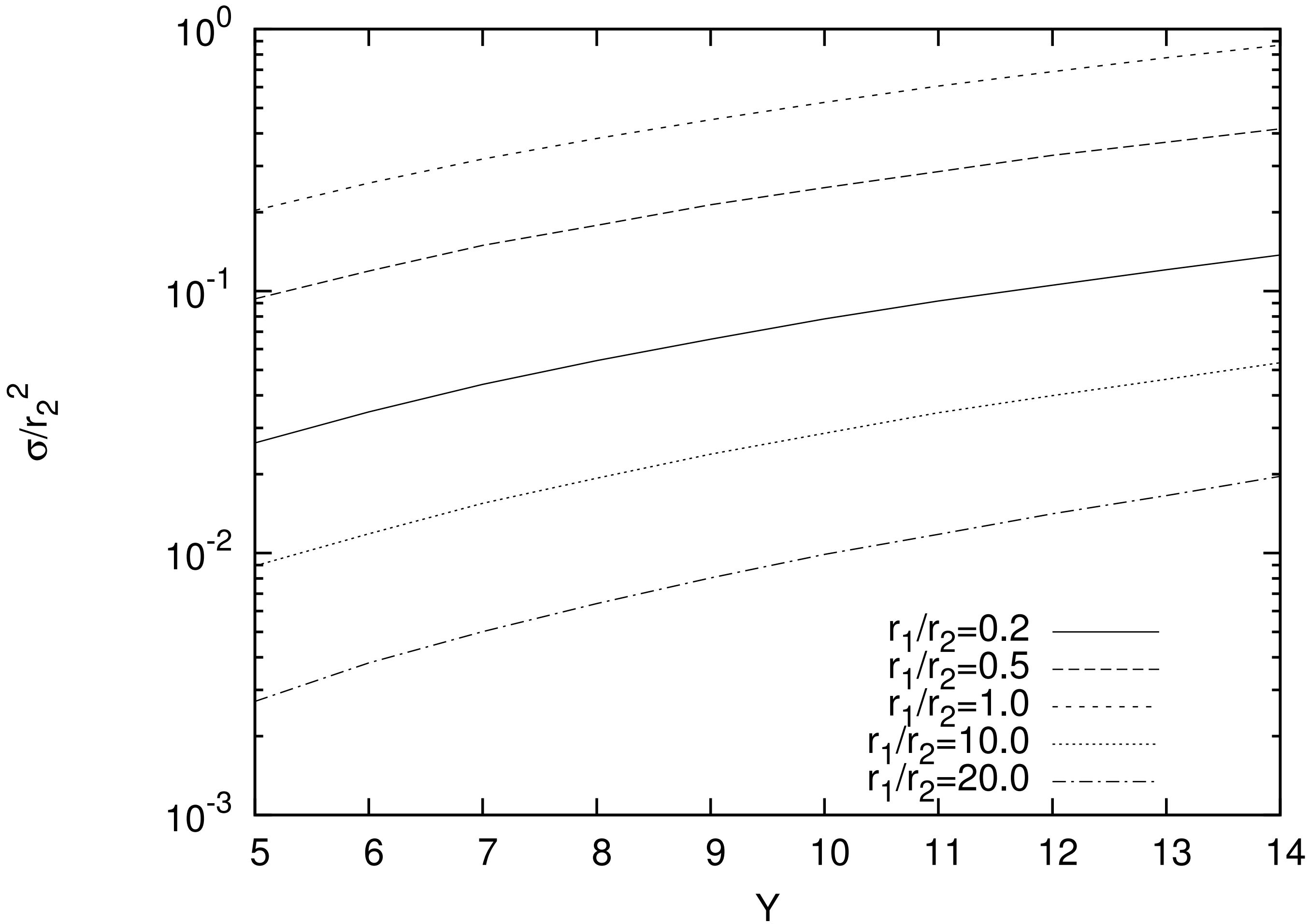
MC simulation based on Mueller's model with an energy-momentum constraint.

Result Without Energy Conservation



Results with Energy Conservation

All calculations performed using fixed coupling constant, $\bar{\alpha} = 0.2$.



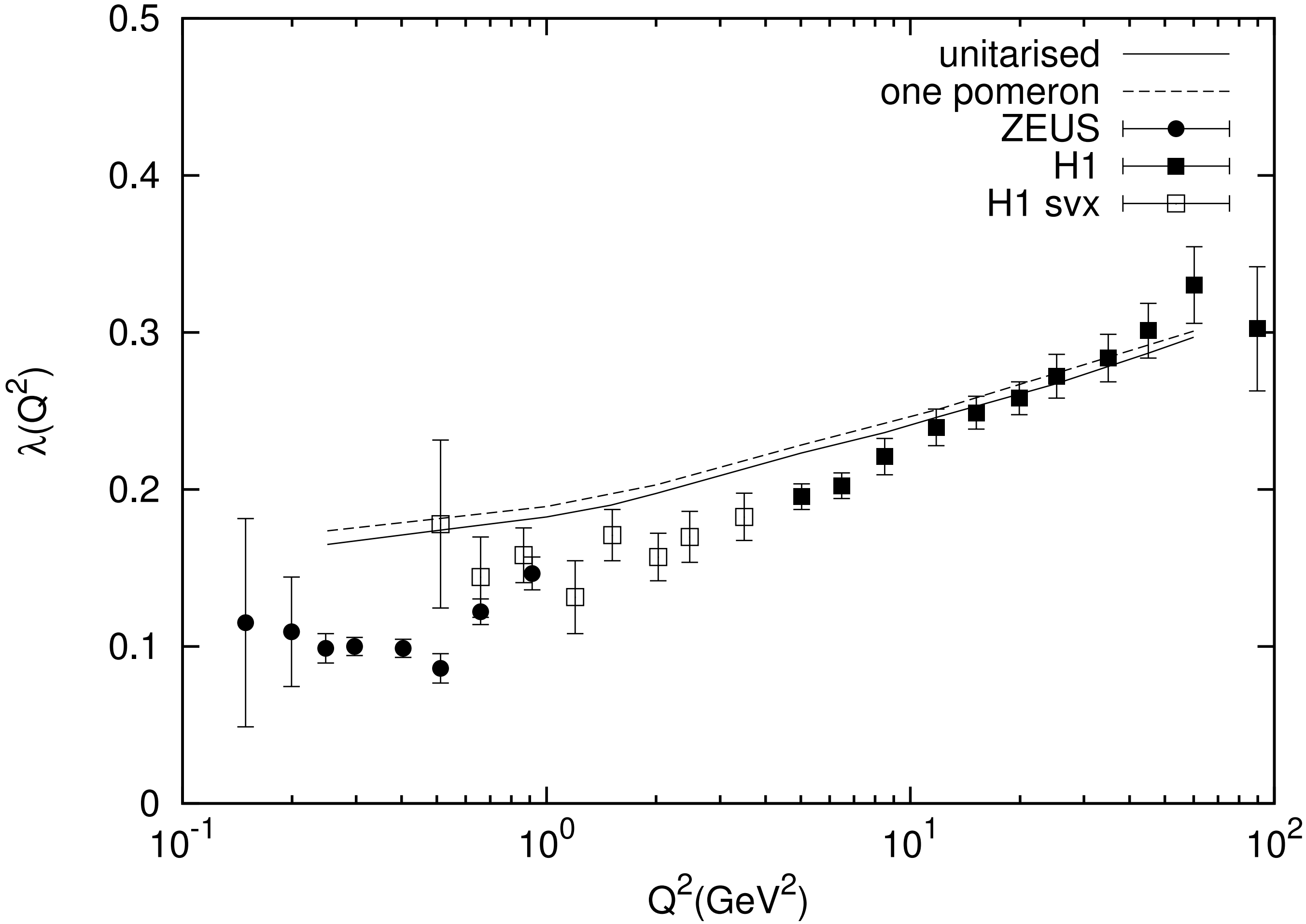
F_2 at HERA

We study only the qualitative behavior. Toy model proton which is given by a collection of colour dipoles distributed with a Gaussian in $\mathbf{r}$ and $\mathbf{b}$.

Identify Q^2 with $1/r^2_{proj}$

λ_{eff} calculated in x-intervals used in the analysis by H1 collaboration.

Results for λ_{eff}



Conclusions

Growth of σ reduced significantly by energy conservation making the inclusion of saturation a small effect. Onset of saturation delayed.

We can obtain, without tuning any parameters, a reasonable qualitative description of F_2 at HERA even without saturation effects.

Several things to improve. Inclusion of true gluon recombinations, running α_s, improving the toy model nucleus etc.