Outline

Dynamik $\Longrightarrow$ Reaktionsdynamik $\Longrightarrow$ Enzymreaktioner $\Longrightarrow$ Cell cyklen

Dynamik

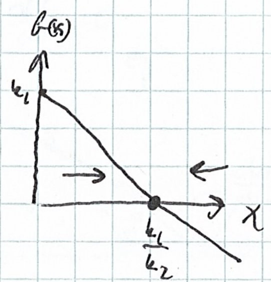
Hur ett system utvecklas över tid

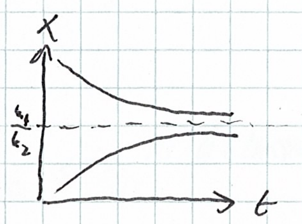
System: $\begin{cases} \text{tillståndsvariabler} \\ \text{växelverkningar} \end{cases}$

ex1

$$\frac{dx}{dt} = k_1 - k_2 x = f(x)$$

Fixpunkter $\Longleftrightarrow$ $f(x) = 0$ $\Longleftrightarrow$ $x^* = \frac{k_1}{k_2}$



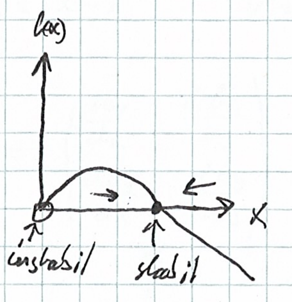


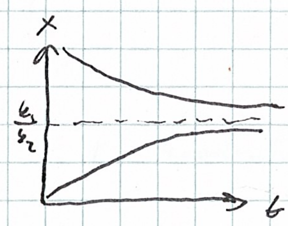
∴ Global stabil fixpunkt

ex2

$$\frac{dx}{dt} = k_1 x - k_2 x^2 = x(k_1 - k_2 x) = f(x)$$

$$f(x) = 0 \Longleftrightarrow \begin{cases} x = 0 \\ \text{el} \\ x = \frac{k_1}{k_2} \end{cases}$$





Också:

$$\left.\frac{df}{dx}\right|_{x^*} = k_1 - 2k_2 x^*$$

$x^* = 0 \Longleftrightarrow \frac{df}{dx} > 0 \Longleftrightarrow$ instabil

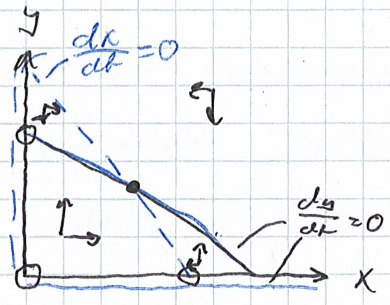
$x^* = \frac{k_1}{k_2} \Longleftrightarrow \frac{df}{dx} < 0 \Longleftrightarrow$ stabil ex 3 Två variabler

$$\frac{dx}{dt} = k_1 x - k_2 x^2 - k_3 xy = f_x(x,y)$$

$$\frac{dy}{dt} = k_4 y - k_5 y^2 - k_3 xy = f_y$$

$$\frac{dx}{dt} = 0 \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{k_1 - k_3 y}{k_2} \end{cases} \qquad \left(y = \frac{k_1 - k_2 x}{k_3} \right)$$

$$\frac{dy}{dt} = 0 \Leftrightarrow \begin{matrix} y = 0 \\ y = \frac{k_4 - k_3 x}{k_5} \end{matrix}$$



$\Leftrightarrow$ Nollkurva

4 fixpunkter där nollkurvor korsar

1 stabil

Linearisera runt fixpunkter

$$u = x - x^* \qquad v = y - y^*$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \frac{df_x}{dx} & \frac{df_x}{dy} \\ \frac{df_y}{dx} & \frac{df_y}{dy} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \text{kvadratiska termer}$$

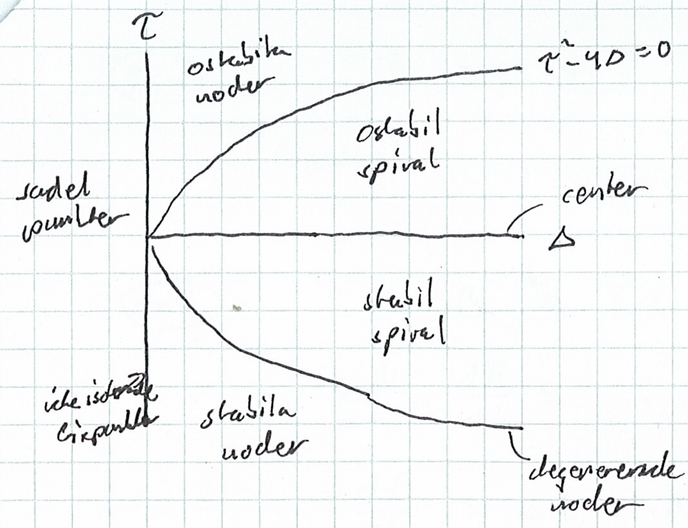
Jacobian, egenvärden bestämmer stabilitet

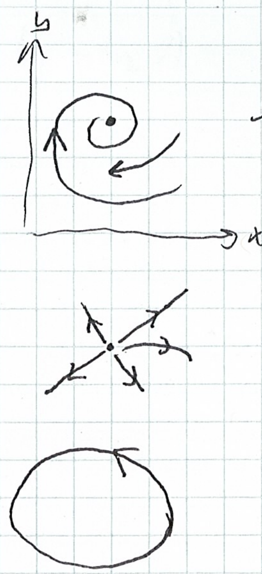
$$\lambda_{1,2} = \frac{1}{2}\left(\tau \pm \sqrt{\tau^2 - 4\Delta} \right)$$

$\tau \equiv$ spåret $= \lambda_1 + \lambda_2$

$\Delta \equiv$ determinanten $= \lambda_1 \lambda_2$

∴ Instabil om något egenvärde har positiv realdel





Kemiska reaktioner

- variabler (koncentrationer) positiva
- parametrar positiva
- aldrig mot oändligheten

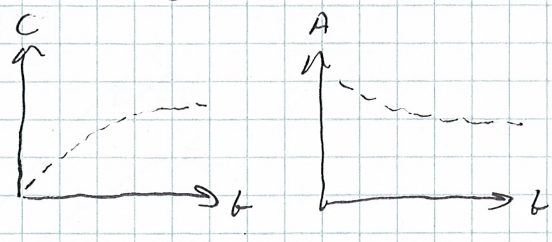
Massverkans lag

Reaktionshastigheten för en (bio)kemisk reaktion är proportionell mot koncentrationerna av molekylerna i reaktionen

ex1 $A + B \underset{(k_2)}{\overset{k_1}{\rightleftharpoons}} C$

Framåt $\frac{dA}{dt} = \frac{dB}{dt} = -k_1 AB = -\frac{dC}{dt}$

Båda $\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = k_2 C - k_1 AB$



Fixpunkt (Jämvikt) $C = \frac{k_1}{k_2} AB$

$\boxed{K = \frac{k_1}{k_2} = \frac{C^*}{A^* B^*}}$ Jämvikts (reaktions) konstant (ex 2) $\phi \underset{k_2}{\overset{k_1}{\rightleftarrows}} A$ $\qquad \frac{dA}{dt} = k_1 - k_2 A$ $\qquad$ Jmf dynamikexempel

"ingenting"

(ex 3) $A + B \xrightarrow{k_1} 2C$

$$\frac{dA}{dt} = \frac{dB}{dt} = -k_1 A \cdot B$$

$$\frac{dC}{dt} = \underline{\underline{2}}\, k_1 A \cdot B$$

(ex 4) Flera reaktioner

$$A \underset{k_{1-}}{\overset{k_{1+}}{\rightleftarrows}} B \underset{k_{2-}}{\overset{k_{2+}}{\rightleftarrows}} C$$

Jämvikt $\quad K_1 = \frac{B}{A} \qquad K_2 = \frac{C}{B} \qquad \frac{C}{A} = \frac{B K_2}{B/K_1} = K_1 \cdot K_2 \equiv K$

∴ Detaljer av mellanliggande steg ändrar inte jämviktskonstanter (koncentrationer)

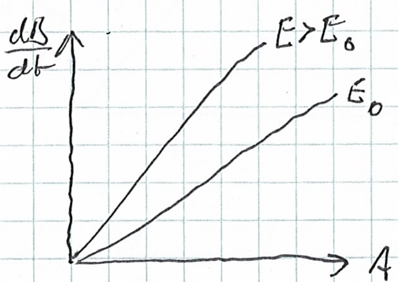
Enzymreaktioner

Enzym: protein (molekyl) som katalyserar långsamma reaktioner utan att försvinna under reaktionen

(ex) $A + E \xrightarrow{k_1} B + E$

$$\frac{dA}{dt} = -\frac{dB}{dt} = -k_1 A \cdot E$$

$$\frac{dE}{dt} = 0$$



∴ Problem: reaktionshastighet $\to \infty$ när $A \to \infty$ även om få enzymmolekyler

Michaelis-Menten

$$S + E \underset{k_2}{\overset{k_1}{\rightleftharpoons}} SE \xrightarrow{k_3} P + E$$

$$\frac{dS}{dt} = -k_1 S \cdot E + k_2 SE$$

$$\frac{dE}{dt} = -k_1 S \cdot E + (k_2 + k_3) SE = -\frac{dSE}{dt}$$

$$\frac{dP}{dt} = k_3 SE$$

Antag:

(i) Första delen av reaktionen snabb $(k_1, k_2 \gg k_3)$

(ii) Konstant mängd enzym $E + SE = E_0$

$$(i) \Rightarrow \frac{dSE}{dt} \approx 0 \Leftrightarrow SE = \frac{k_1 S \cdot E}{k_2 + k_3} \underset{(ii)}{=} \frac{k_1 S(E_0 - SE)}{k_2 + k_3}$$

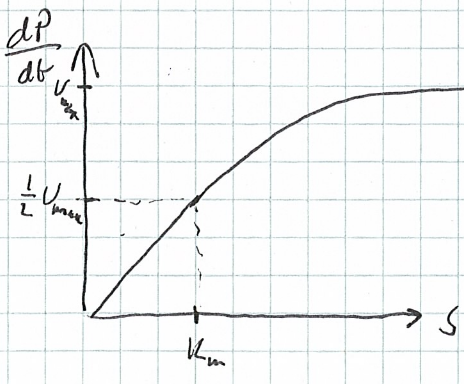
$$\underbrace{\frac{k_2 + k_3}{k_1}}_{K_m} SE = S E_0 - S \cdot SE$$

$$SE(K_m + S) = S \cdot E_0$$

$$SE = \frac{S E_0}{K_m + S}$$

$$\boxed{\frac{dP}{dt} = k_3 SE = \frac{V_{max} S}{K_m + S}}$$

$$\begin{pmatrix} V_{max} = k_3 E_0 \\ K_m = \frac{k_2 + k_3}{k_1} \end{pmatrix}$$



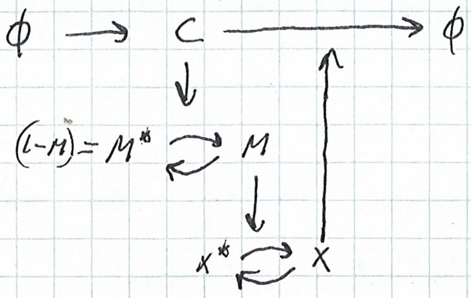
Saturerad ☺

Om E_0 dynamisk variabel

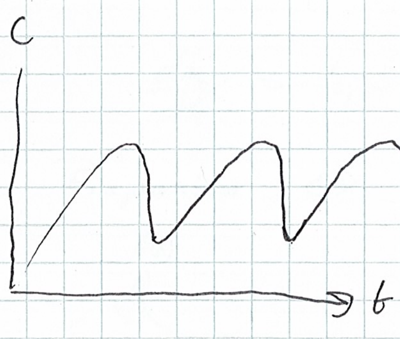
$$\frac{dP}{dt} = \frac{V_m E_0 \cdot S}{K_m + S}$$

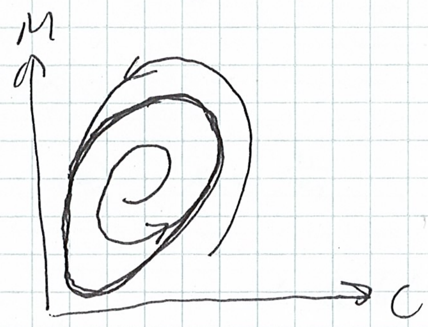
ex Goldbeter cellcykel

Goldbeter slide



$$\begin{cases} \dfrac{dC}{dt} = v_i - v_d \dfrac{XC}{K_d + C} - k_d C \\[2ex] \dfrac{dM}{dt} = V_1 \dfrac{C(1-M)}{K_1 + (1-M)} - V_2 \dfrac{M}{K_2 + M} \\[2ex] \dfrac{dX}{dt} = V_3 \dfrac{M(1-X)}{K_3 + (1-X)} - V_4 \dfrac{X}{K_4 + X} \end{cases}$$





Crowded cell slide