Lund University

Towards $N_c = 3$ parton showers

- Brief parton shower overview
- Why N_c suppressed terms may be important
- Some $N_c = 3$ shower challenges
- First results from an $SU(3)$, rather than $SU(\infty)$ parton shower, in collaboration with Simon Plätzer
- Remaining shower and event generator challenges

Cambridge
October 29, 2013
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An ordinary parton shower

- Treats:

$$p \quad \otimes \quad f \ \otimes \text{ spin average } \otimes N_c \to \infty \text{ color}$$

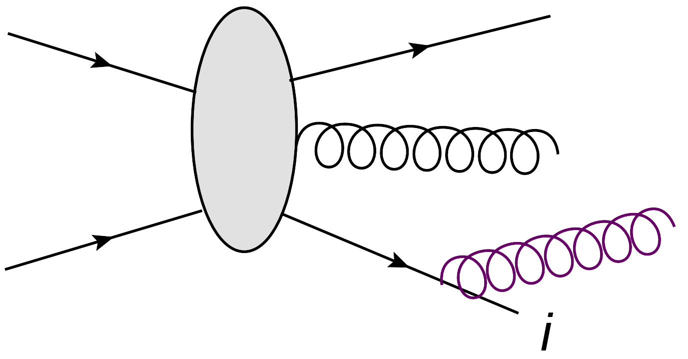
- Emits one particle at the time
- Assumes an ordering variable like $k_\perp$, virtuality, angle

$$\Delta(t) = \exp(-\int_{t_0}^{t} \frac{dt'}{t'} \alpha_{\mathrm{s}}(t') \int ...)$$

- Resums large logs which compensate for the smallness of α_{s}
- is a Markov process at the $|A|^2$ level
  (the next step depends on the state, but not on the history)

The first generation parton shower

- A parton can be seen as emitted from one other parton using pure $1 \to 2$ splitting (JETSET)



- Resums the collinear splitting probability using DGLAP splitting functions

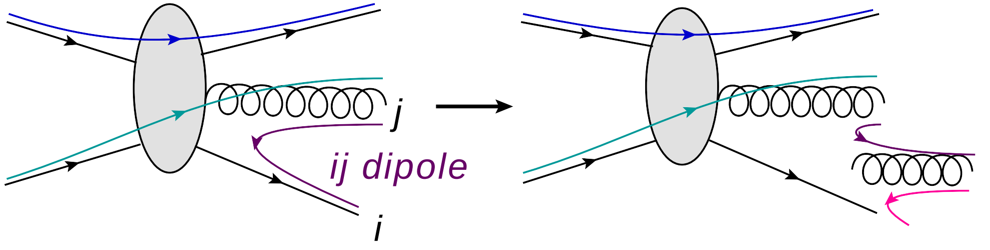
$$\Delta_k(t) = \exp(-\sum_i \int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t') \int \frac{dz}{2\pi} P_{ik})$$

A second generation parton shower

- Resums also the softly enhanced radiation probabilities in the $N_c \to \infty$ limit

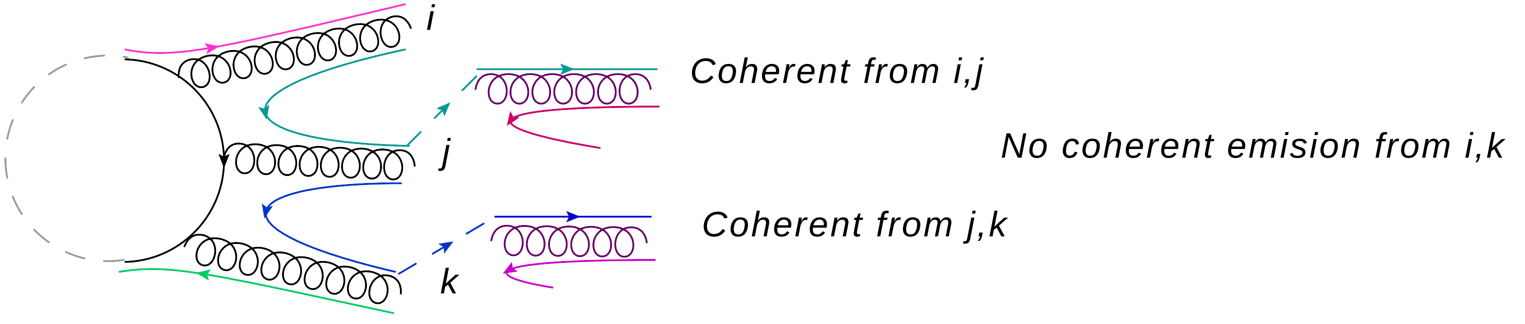
$$\Delta_k(t) = \exp(-\frac{2}{\pi}\sum_{\text{dipoles i,j(i)}}\int_{t_0}^{t}\frac{dt'}{t'}\alpha_s(t')\int\frac{dy d\phi}{2\pi}\frac{k_T^2 p_i.p_j}{2\, p_i.k\ p_j.k})$$

- In the soft limit a parton can be seen as being emitted coherently from a pair of color connected partons, 2 $\to$ 3 splitting from a color point of view



A dipole shower in the large N_c limit

- A dipole shower can easily be thought of in the $N_c \to \infty$ limit



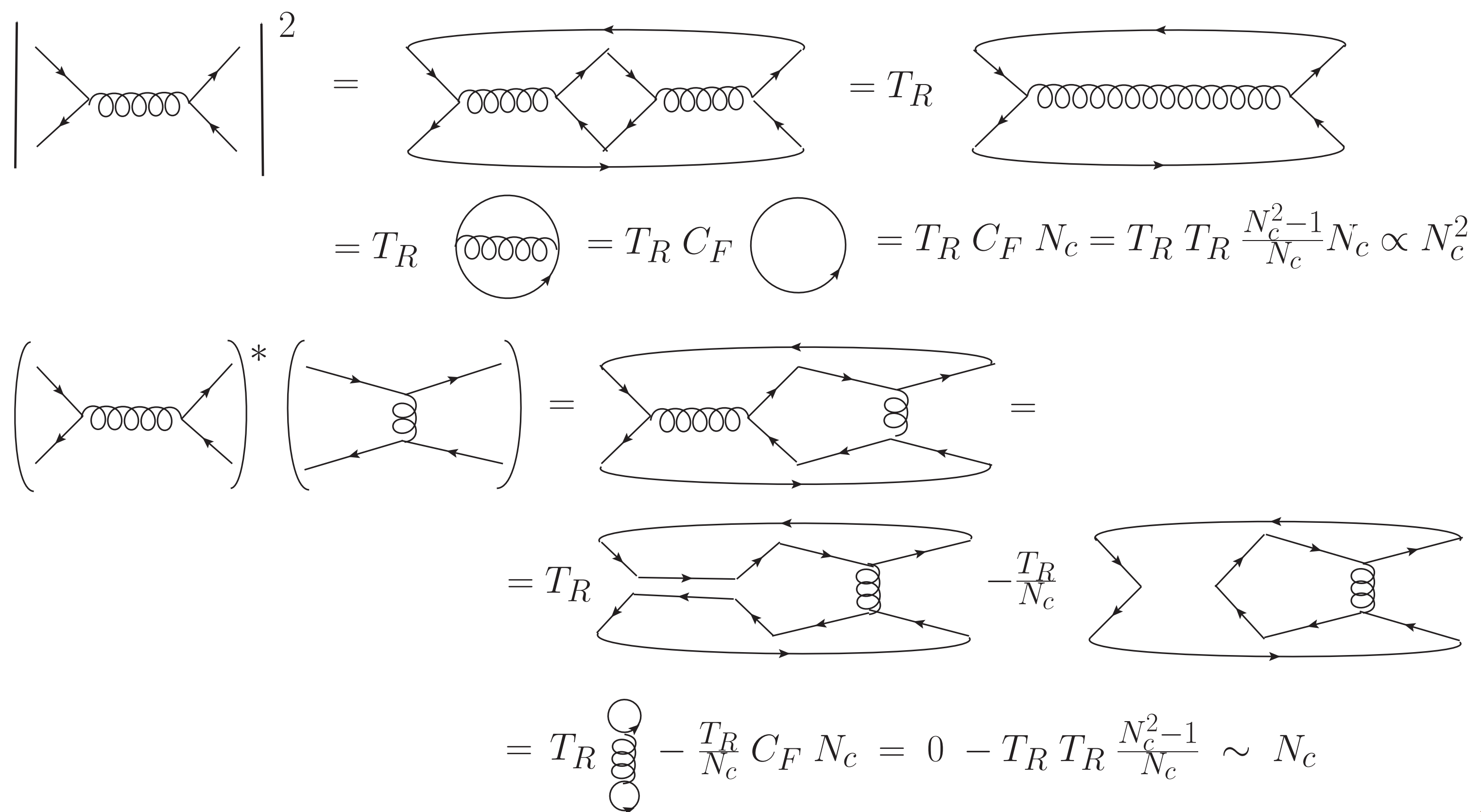
- In this limit only "color neighbors" radiate, i.e., only neighboring partons on the quark-lines in the basis radiate
- All standard parton showers (Pythia, Herwig, Sherpa) work in this $N_c \to \infty$ limit

Why worry about N_c suppressed terms?

- " Non-leading color terms are suppressed by $1/N_c^2$ "
  (not quite true, but assume for now that it is)
- If one starts with only one color flow the number of leading color emission possibilities grows just like $\sim N_{\text{partons}}$
- The total number of possibilities for coherent emission grows as $\sim N^2_{\text{partons}}$ (taking any pair)
- If non-leading terms always were N_c^2 suppressed, the relative importance can grow like $\sim (N_{\text{partons}})/N_c^2$
  (slower with random averaging)
- In general one does not start with only one color flow
  $\rightarrow$ relative importance of suppressed terms can be larger, up to $(N_{q\bar{q}} + N_g)!/N_c^2$

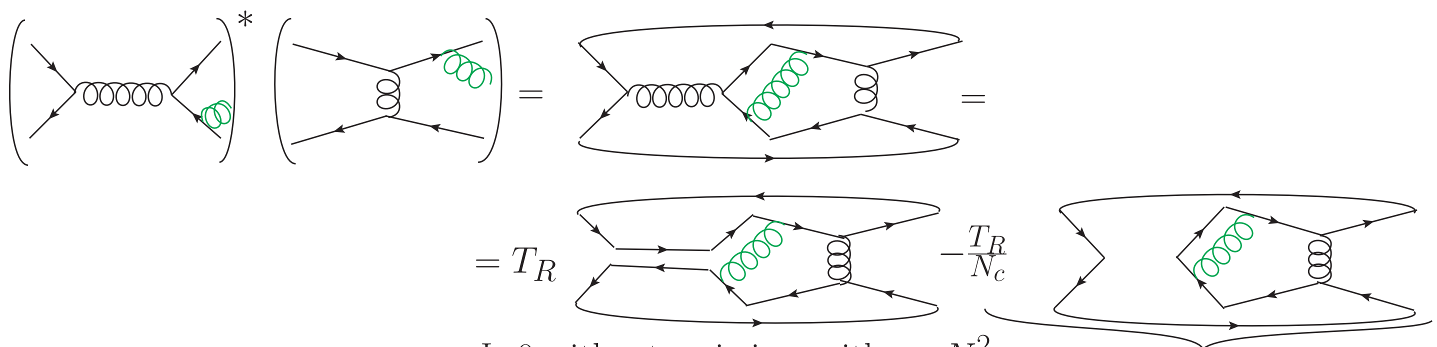
$1/N_c$ suppressed terms

That non-leading color terms are suppressed by $1/N_c^2$, is guaranteed only for same order α_s diagrams with only gluons ('t Hooft 1973)



$1/N_c$-suppressed terms

For a parton shower there may also be terms which only are suppressed by one power of N_c



Is 0 without emission, with $\sim N_c^2$ did not enter in any form, genuine "shower" contribution

Is $\sim N_c$ without emission, with $\sim N_c^2$ "included" in shower, contribution from hard process

The leading N_c contribution scales as N_c^2 before emission and N_c^3 after

- “ Non-leading color terms **tend to be** suppressed by $1/N_c^2$ ”
  counter examples exist
- Is true for same order α_s diagrams with only gluons ('t Hooft 1973)
- A parton shower is an all order (Sudakov) exponentiation

$$\Delta(t) = \exp(-\int_{t_0}^{t} \frac{dt'}{t'} \alpha_s(t) \int ...)$$

- Certainly not only one power in α_s is needed

Some rescuing mechanisms?

- In the collinearly (rather than softly) enhanced regions, the emitted parton can be seen as coming from only one parton and the color structure is trivial $\rightarrow$ no need for N_c suppressed terms
- Random averaging:
  The suppressed terms sometimes contribute positively to the cross section, and sometimes negatively. Perhaps they tend to cancel quicker than expected? (Via correlations in emission?)
- α_s suppression: $1/N_c^1$ suppressed terms tend to also be associated with powers of α_s^2, but remember:
  Large logs accompany α_s, this is why we need resummation

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Yes, several

Major challenges for SU(3) showers

Three major challenges dealt with so far

- Evolution with amplitude information, we have a Markov process at the amplitude level (more later)
- Negative contributions to radiation probability, "negative splitting kernels", treated using interleaved veto/competition algorithm (S. Plätzer & M. S., EPJ Plus 127 (2012) 26)
- Keeping track of the color structure for an arbitrary number of partons (next)

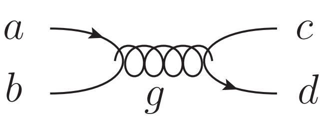
Dealing with color space

- We never observe individual colors
  $\rightarrow$ we are only interested in color summed/averaged quantities
- For given external partons, the color space is a finite dimensional vector space equipped with a scalar product

$$\langle A, B\rangle = \sum_{a,b,c,\ldots} (A_{a,b,c,\ldots})^* B_{a,b,c,\ldots}$$

Example: If

$$A = \sum_g (t^g)^a{}_b (t^g)^c{}_d =$$



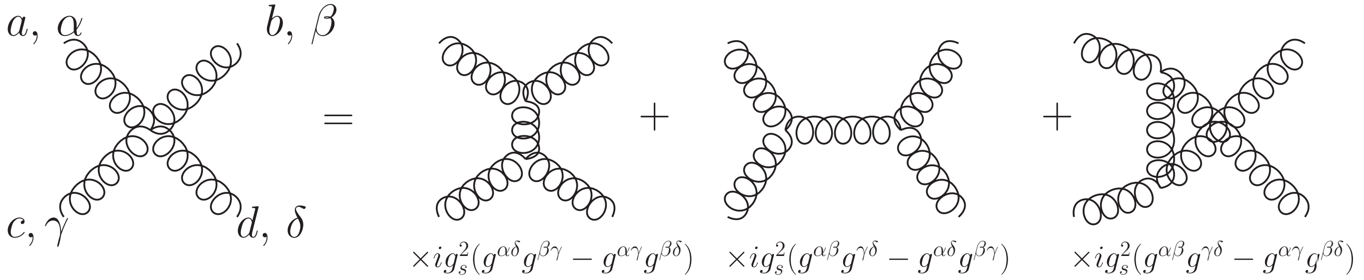
,

then $\langle A|A\rangle = \sum_{a,b,c,d,g,h} (t^h)^b{}_a (t^h)^d{}_c (t^g)^a{}_b (t^g)^c{}_d$

- One way of dealing with color space is to just square the amplitudes one by one as they are encountered
- Alternatively, we may use a basis (spanning set)

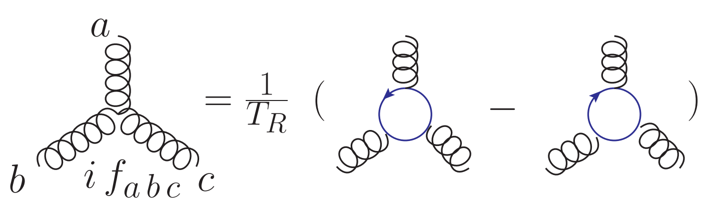
The standard treatment: Trace bases

- Every 4g vertex can be replaced by 3g vertices:

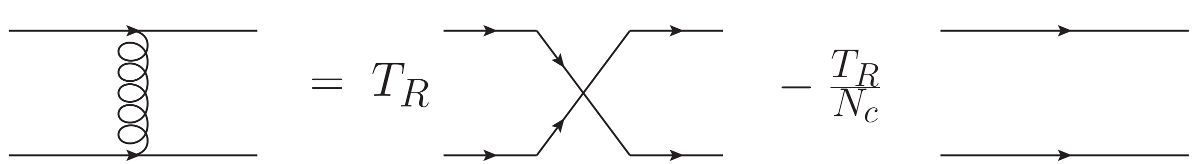


(read counter clockwise)

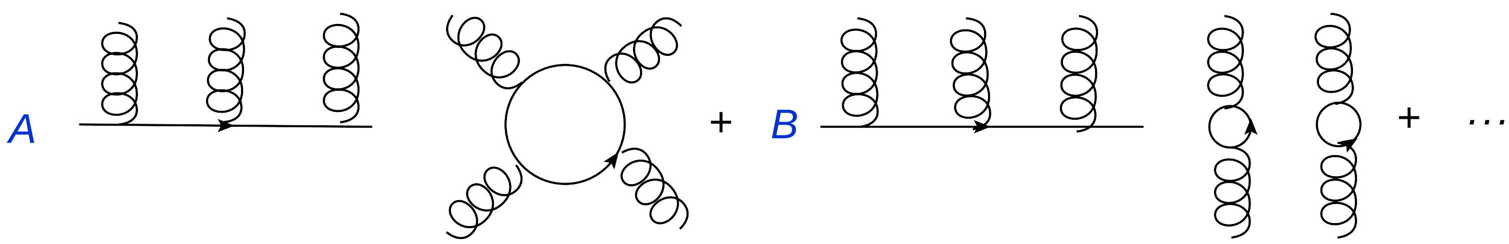
- Every 3g vertex can be replaced using:



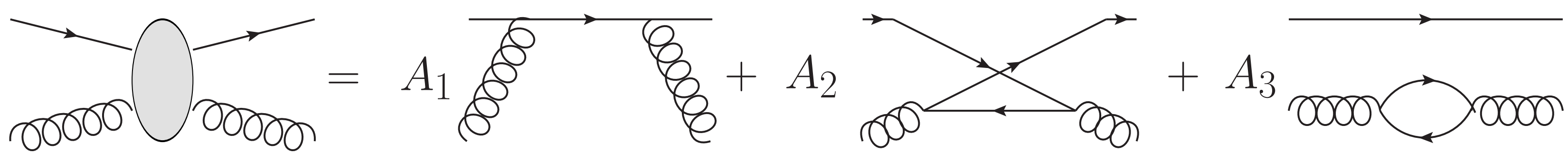
- After this every internal gluon can be removed using:



- This can be applied to any QCD amplitude, tree level or beyond
- In general an amplitude can be written as linear combination of different color structures, like



- For example for 2 (incoming + outgoing) gluons and one $q\bar{q}$ pair

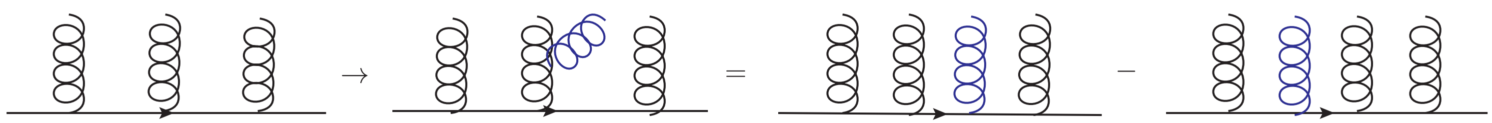


(an incoming quark is the same as an outgoing anti-quark)

The above type of color structures can be used as a spanning set, a trace basis. (Technically the set of all such color structures is in general overcomplete, so it is rather a spanning set.)

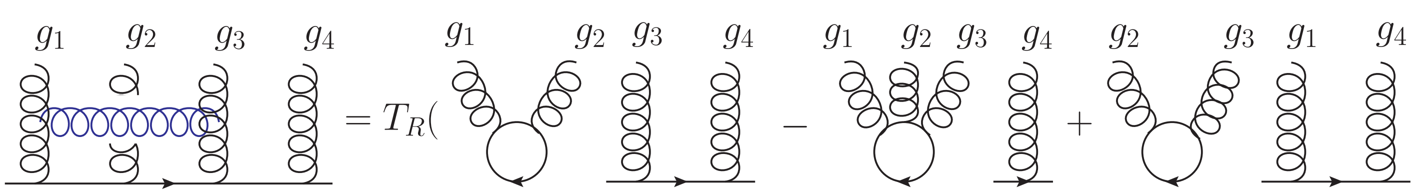
These bases have some nice properties

- The effect of gluon emission is easily described:



Convention: + when inserting after, minus when inserting before.

- So is the effect of gluon exchange:



Convention: + when inserting after, - when inserting before

ColorFull

For the purpose of treating a general QCD color structure I have written a C++ color algebra code, ColorFull, which:

- Is used in the color shower with Simon Plätzer
- Automatically creates a "trace basis" for any number and kind of partons, and to arbitrary order in α_s
- Squares color amplitudes
- Describes the effect of gluon emission, and gluon exchange
- Interfaces to Herwig++ ($\geq$ 2.7) via Simon's Matchbox code
- Is used for NLO electroweak Higgs + 3 jet production, in collaboration with Francisco Campanario, Terrance Figy and Simon Plätzer, arXiv:1308.2932, accepted for publication in PRL

ColorFull is now publicly available in a **pre-release** version at `colorfull.hepforge.org`.

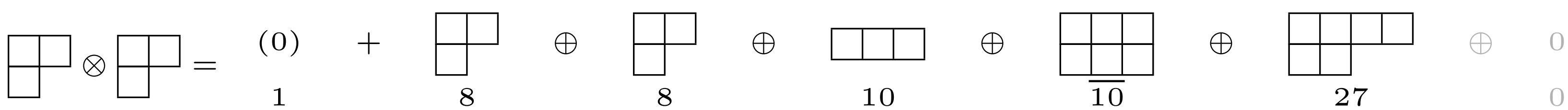
But...

- this type of basis is non-orthogonal and overcomplete
  (for more than a few partons)
- ... and the number of basis vectors grows as a factorial in $N_g + N_{q\overline{q}}$
  $\rightarrow$ when squaring amplitudes we run into a factorial square scaling
- Hard to go beyond $q\overline{q} + 7$ gluons
- Would be nice with minimal orthogonal basis

Minimal orthogonal bases for color spaces

In collaboration with Stefan Keppeler (Tübingen)

- Want orthogonal minimal basis for color space
- Basis vectors can be enumerated using Young tableaux multiplication, for example for $gg \to gg$



and constructed if projection operators are known

- The problem is the construction of the corresponding projection operators; the Young-tableaux operate with "quark-units" but the physical particles include anti-quarks and gluons

- One may think that the problem of constructing group theory based multiplet bases should have been solved a long time ago
- The $2g \to 2g$ case was solved in the 60's ($N_c = 3$)
- However, until recently only a few cases had been dealt with, those for which (loosely speaking) nothing more complicated than two gluon projection operators is needed
- About one year ago me and Stefan Keppeler presented a general recipe for constructing gluon projection operators. From these we also know how to construct orthogonal bases for any number and kind of partons, JHEP09(2012)124, arXiv:1207.0609

- For many partons the size of the vector space is much smaller for $N_c = 3$ (exponential), compared to for $N_c \to \infty$ (factorial)

Case	Vectors $N_c = 3$	Vectors, general case
4 gluons	8	9
6 gluons	145	265
8 gluons	3 598	14 833
10 gluons	107 160	1 334 961

Number of basis vectors for $N_g \to N_g$ gluons
without imposing vectors to appear in charge conjugation invariant combinations

Conclusions for the color space

- One strategy is to use "trace bases". These are not orthogonal, but have other advantages
- ColorFull is a C++ tool for treating the color structure in trace bases, `colorfull.hepforge.org`
- Alternatively one may want to use orthogonal multiplet bases
  S. Keppeler & M.S., JHEP09(2012)124, arXiv:1207.0609
- Number of basis vectors then grows only exponentially for $N_c = 3$
- This has the potential to very significantly speed up exact calculations in the color space of $SU(N_c)$
- However, in order to use this in an optimized way, we need to understand how to sort QCD amplitudes in this basis in an efficient way (work in progress, but I'm optimistic)

Basics of our shower

S. Plätzer & M.S., JHEP 07(2012)042, arXiv:1201.0260

- Built on the Catani-Seymour dipole factorization
  (S.Plätzer & S. Gieseke, JHEP 1101, 024 (2011) & 1109.6256)
- Parton $\tilde{ij}$ splitting to partons i and j, and parton $\tilde{k}$ absorbs the longitudinal recoil such that all partons remain on shell

$$|\mathcal{M}_{n+1}(...,p_i,...,p_j,...,p_k,...)|^2 \approx$$
$$\sum_{k\neq i,j} \frac{1}{2p_i\cdot p_j} \langle \mathcal{M}_n(...,p_{\tilde{ij}},...,p_{\tilde{k}},...)|\mathbf{V}_{ij,k}(p_i,p_j,p_k)|\mathcal{M}_n(...,p_{\tilde{ij}},...,p_{\tilde{k}},...)\rangle$$

- In a standard parton shower parton $\tilde{ij}$ and $\tilde{k}$ would have to be "color connected",

$$\mathbf{V}_{ij,k} = -8\pi\alpha_s V_{ij,k} \frac{\mathbf{T}_{\tilde{ij}}\cdot\mathbf{T}_{\tilde{k}}}{\mathbf{T}^2_{\tilde{ij}}} \rightarrow 8\pi\alpha_s \frac{V_{ij,k}}{1+\delta_{\tilde{ij}}}\delta(\tilde{ij},\tilde{k} \text{ color connected})$$

we keep all pairs ($\delta_{\tilde{ij}}=1$ for gluon, 0 else, $\mathbf{T}^2_{\tilde{ij}}$ is a convention)

For the emission probability this means that:

$$dP_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2}\frac{\langle \mathcal{M}_n|\mathbf{T}_{\tilde{ij}}\cdot\mathbf{T}_{\tilde{k}}|\mathcal{M}_n\rangle}{|\mathcal{M}_n|^2}$$

rather than

$$dP_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z)\frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times \frac{\delta(\tilde{ij}, \tilde{k}\ \text{color connected})}{1+\delta_{\tilde{ij}}}$$

The splitting kernels read:

$$\begin{aligned} V_{qg,k}(p_i, p_j, p_k) &= C_F\left(\frac{2(1-z)}{(1-z)^2 + p_\perp^2/s_{ijk}} - (1+z)\right) \\ V_{gg,k}(p_i, p_j, p_k) &= 2C_A\left(\frac{1-z}{(1-z)^2 + p_\perp^2/s_{ijk}} + \frac{z}{z^2 + p_\perp^2/s_{ijk}} - 2 + z(1-z)\right) \end{aligned}$$

Evolution with amplitude information

- Assume we have a basis for the color space

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \quad \leftrightarrow \quad \mathcal{M}_n = (c_{n,1}, ..., c_{n,d_n})^T$$

  (this basis need not be orthogonal)
- $|\mathcal{M}_n\rangle$ is known for the hard process
- How do we get $|\mathcal{M}_{n+1}\rangle$ after emission?

- Observe that

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

(where S_n is the color scalar product matrix)
and

$$\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n} \mathcal{M}_n \mathcal{M}_n^\dagger T_{\tilde{ij},n}^\dagger\right)$$

- Use an "amplitude matrix" $M_n = \mathcal{M}_n \mathcal{M}_n^\dagger$ as basic object

$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}_{\tilde{ij}}^2} \, T_{\tilde{k},n} M_n T_{\tilde{ij},n}^\dagger$$

where

$$M_{\text{hard}} = \mathcal{M}_{\text{hard}} \mathcal{M}_{\text{hard}}^\dagger$$

Our current implementation

A proof of concept:

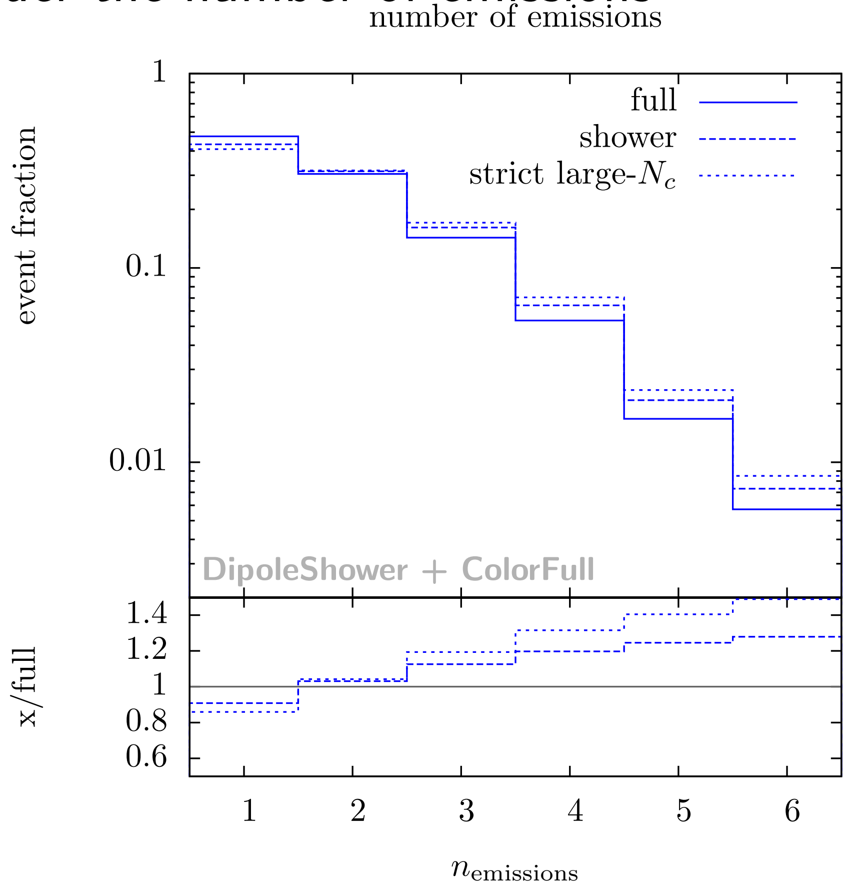
- $e^+e^- \to$ jets, a LEP-like setting
- Fixed $\alpha_s = 0.112$
- Up to 6 gluons, only gluon emission, $g \to q\bar{q}$ is suppressed anyway, and there is no non-trivial color structure
- No hadronization, we don't want to spoil our $N_c = 3$ parton shower by attaching an $N_c \to \infty$ hadronization model. Also, comparing showers in a fair way, would require retuning the hadronized $N_c = 3$ shower
- No "virtual" corrections, i.e. no color rearrangement without radiation, no Coulomb gluons

Three different treatments of color space

- Full, exact SU(3) treatment, all color correlations
- Shower, resembles standard showers, C_F for gluon emission off quarks is exact but non-trivial color suppressed terms are dropped
- Strict large-N_c, all N_c suppressed terms dropped, $C_F = 4/3 \to 3/2$ $(T_R = 1/2)$

Results: Number of emissions

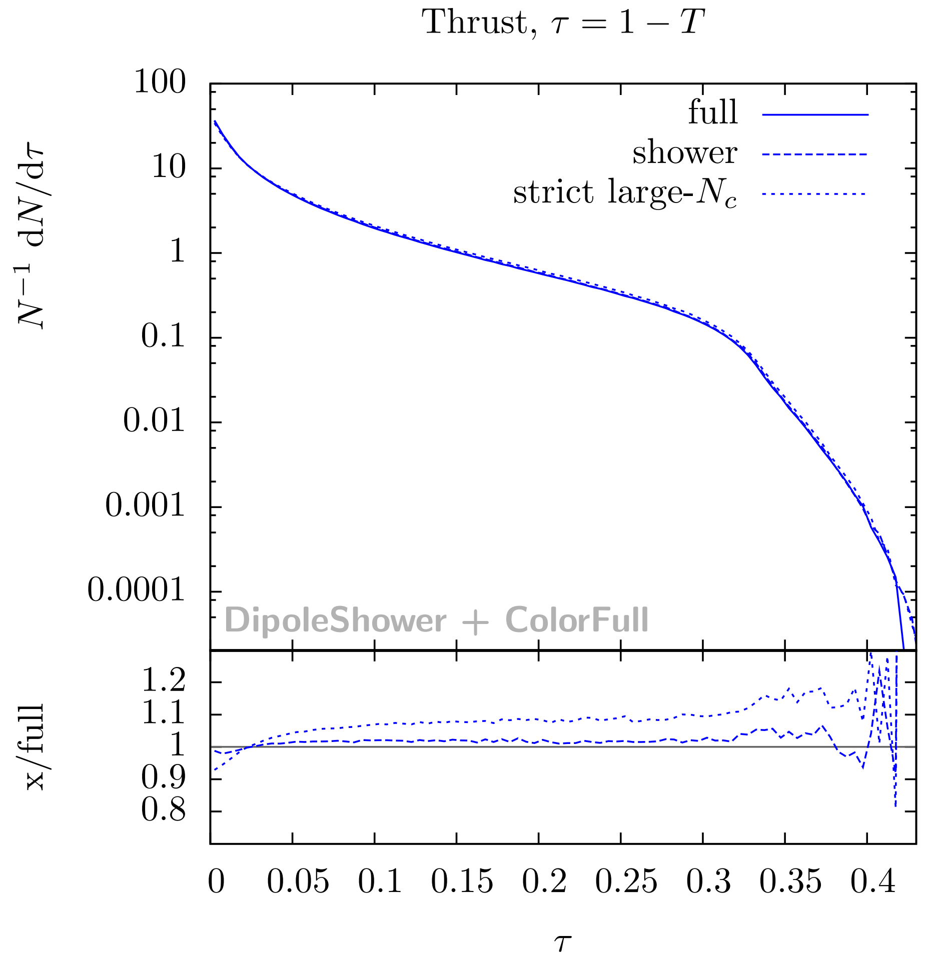
First, simply consider the number of emissions



... this is not an observable, but it is a genuine uncertainty on the number of emissions in the perturbative part of a parton shower

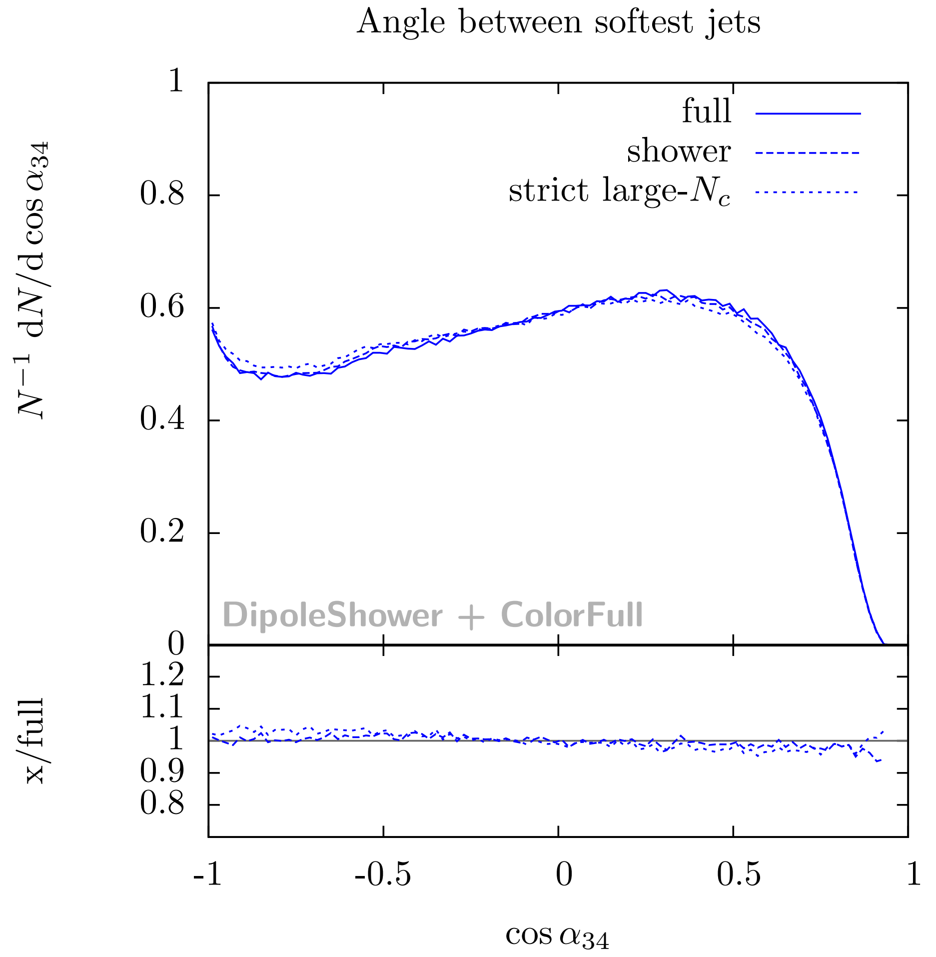
Results: Thrust

For standard observables small effects, here thrust $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$



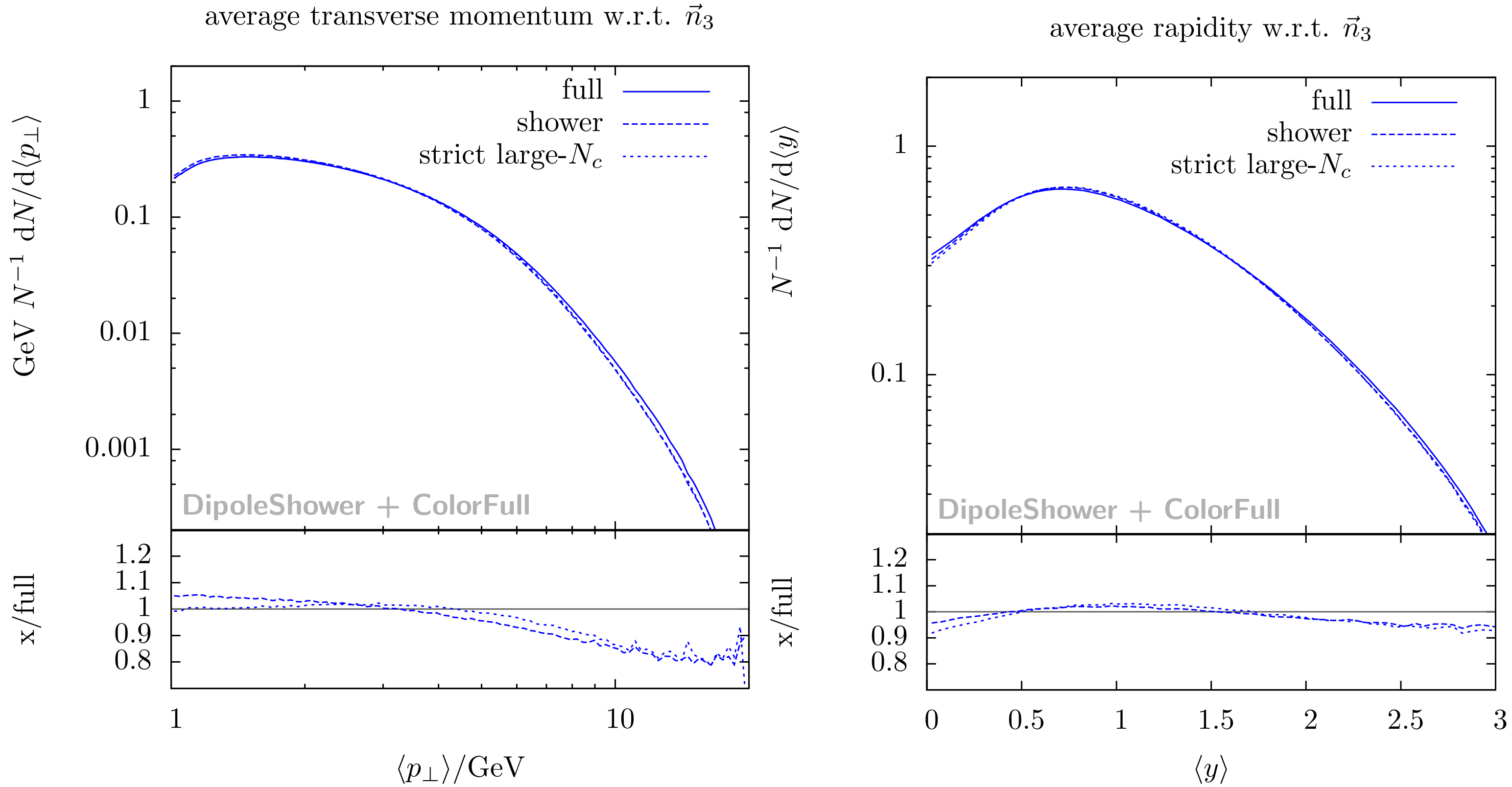
Results: Angular distribution

Cosine of angle between third and fourth jet



Results: Some tailored observables

For tailored observables we find larger differences



Average transverse momentum and rapidity of softer particles with respect to the thrust axis defined by the three hardest partons

Concluded for our LEP-like shower:

- For standard observables we find small deviations for LEP, of order a few percent
- Leading N_c was probably a very good approximation for standard observables at LEP
- For tailored observables we find larger differences $\approx 20\%$
- Keeping C_F to its $N_c = 3$ value $(4/3)$ (as is done in standard showers), rather than $3/2$, tends to improve the approximation $(T_R = 1/2)$
- At the LHC we have many more colored particles, so (many more)^2 possible color suppressed interference terms

Different sources of N_c-suppressed terms

- In a tree level parton shower (no virtual exchange, only emission), N_c-suppressed terms are ignored
  $\rightarrow$ one source of ignored $1/N_c^{(2)}$ (included)
- At loop level, another source of suppressed terms come from virtual gluon exchanges which rearrange the color structure
  $\rightarrow$ The color rearranging terms tend to be suppressed, but in

$$\exp(-\int_{t_0}^{t} \frac{dt'}{t'}(\text{moderate} + \text{small}))$$

  the small number is not irrelevant when $\int_{t_0}^{t} \frac{dt'}{t'}$ is large!
  $\rightarrow$ different source of N_c-suppressed terms (missing)
- Hadronization: How does cluster or sting fragmentaion work for $N_c = 3$, starting from a "quantum shower" (new modeling needed)

Other remaining challenges:

- Hadronic initial state: We should move to LHC phenomenology
- Gluon splitting to $q\bar{q}$ (nothing conceptually new)
- The color structure, with the current implementation we cannot go beyond $\sim$ 8 gluons $+q\bar{q}$-pairs. Should we use another basis? Should we approximate the color structure by keeping only some color suppressed terms?

Conclusions and outlook

- We have written a first $N_c = 3$ parton shower
- To accomplish this we had to deal with several new challenges:
  - complicated color space, ColorFull, `colorfull.hepforge.org`
  - negative probabilities, (EPJ Plus 127 (2012) 26)
  - quantum evolution, (JHEP 07(2012)042, arXiv:1201.0260)
- However *many* challenges still remain:
  - initial state hadrons
  - virtual gluon exchange
  - better treatment of color space
  - hadronization

Thank you for your attention!

Backup: The Sudakov decomposition

In each splitting a parton $\tilde{ij}$ splits into i and j whereas a spectator $\tilde{k}$ takes up the longitudinal recoil

$$p_i = z p_{\tilde{ij}} + \frac{p_\perp^2}{z s_{ijk}} p_{\tilde{k}} + k_\perp \tag{1}$$

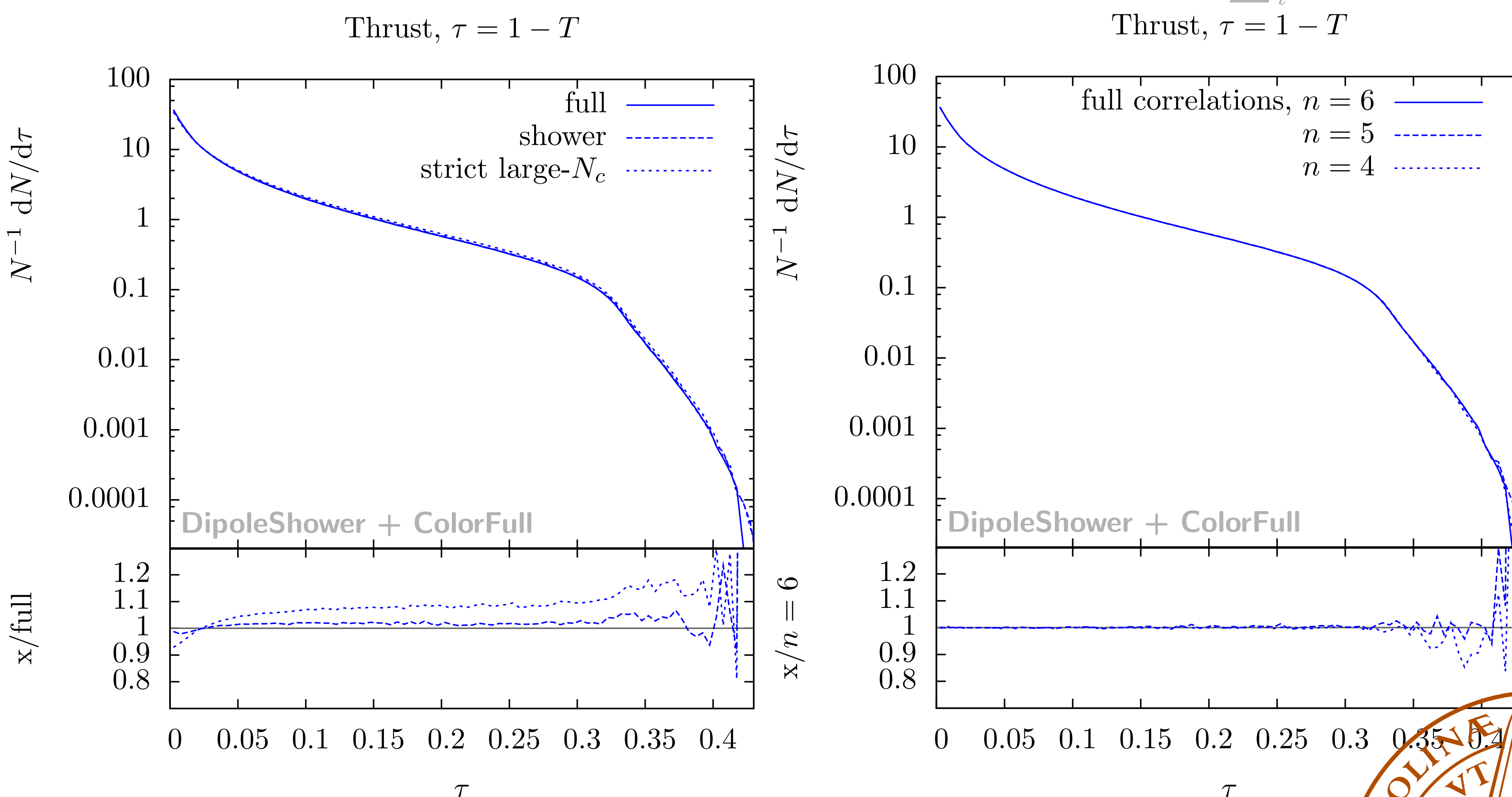
$$p_j = (1-z) p_{\tilde{ij}} + \frac{p_\perp^2}{(1-z) s_{ijk}} p_{\tilde{k}} - k_\perp \tag{2}$$

$$p_k = \left(1 - \frac{p_\perp^2}{z(1-z) s_{ijk}}\right) p_{\tilde{k}} \ , \tag{3}$$

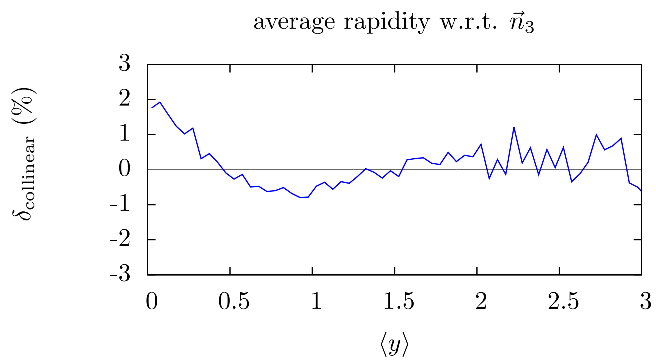
with $p_{\tilde{ij}}^2 = p_{\tilde{k}}^2 = 0$, a space like transverse momentum $k_\perp$ with $k_\perp^2 = -p_\perp^2$ and $k_\perp \cdot p_{\tilde{ij}} = k_\perp \cdot p_{\tilde{k}} = 0$. With this parametrization we also have $s_{ijk} = (p_i + p_j + p_k)^2 = (p_{\tilde{ij}} + p_{\tilde{k}})^2$.

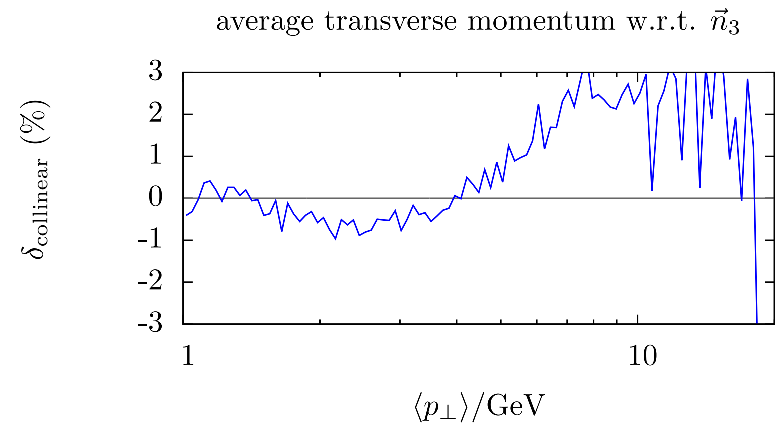
Backup: Thrust

For standard observables small effects, here thrust $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$



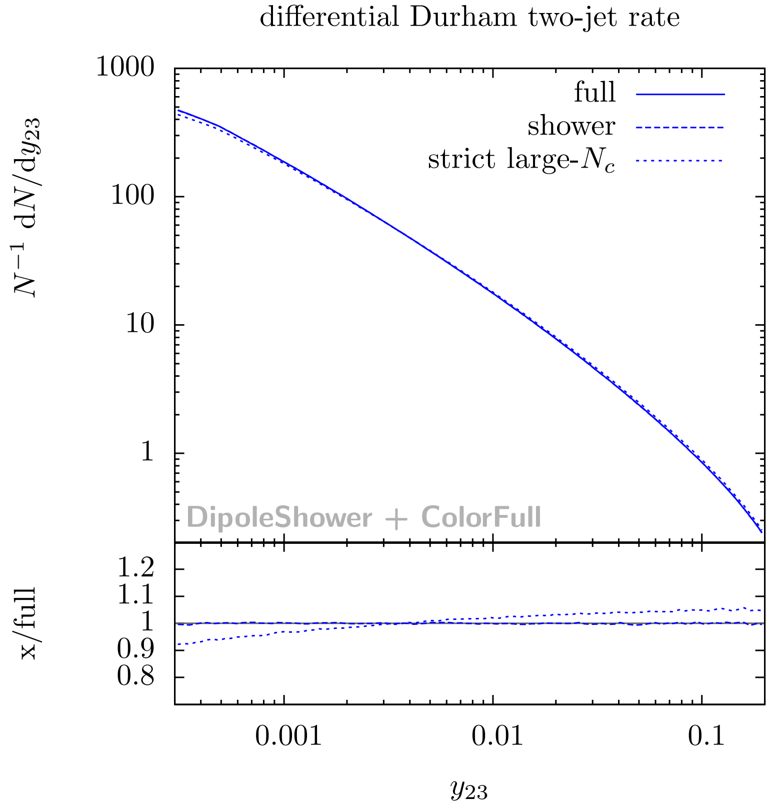
Backup: Importance of $g \to q\bar{q}$ splitting

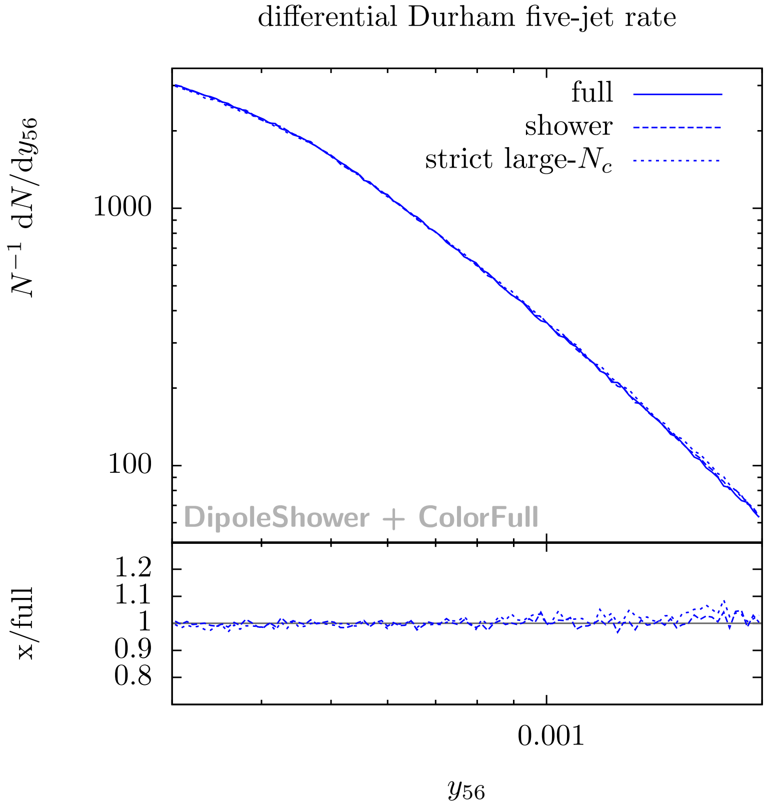




Influence on average transverse momentum and rapidity w.r.t. the thrust axis defined by the three hardest patrons

Backup: Jet separation



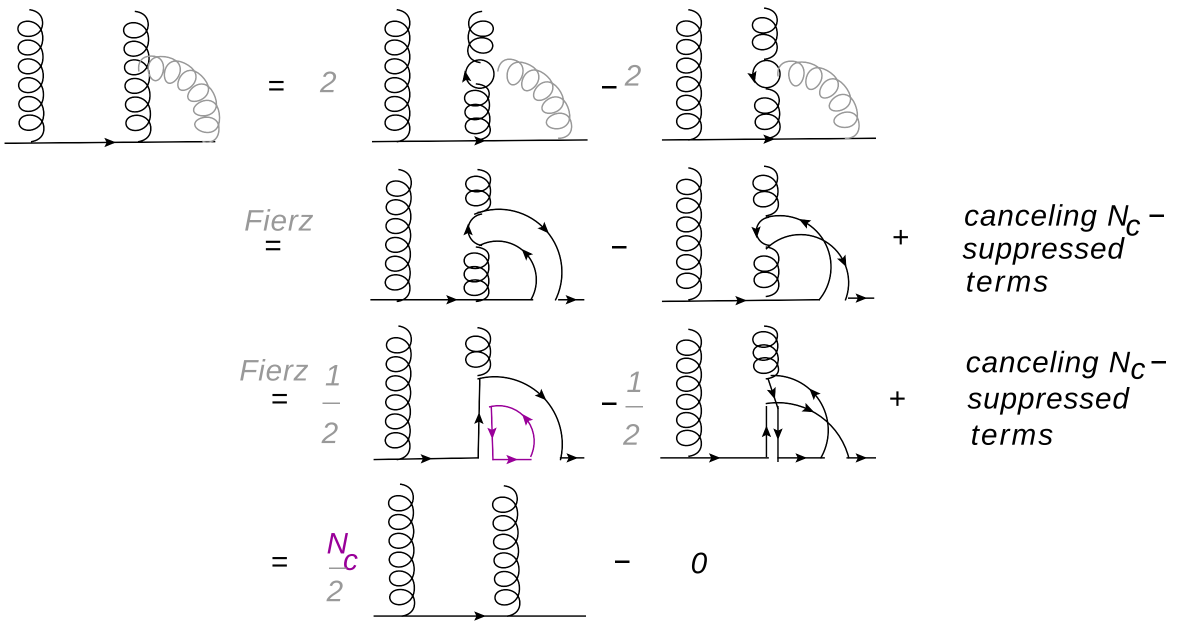


Jet separation between 2nd and 3rd jet, and 5th and 6th jet

$y = 2\,\mathrm{min}(E_i^2, E_j^2)(1-\cos\theta_{ij})/s$

Backup: Gluon exchange

A gluon exchange in this basis "directly" i.e. without using scalar products gives back a linear combination of (at most 4) basis tensors



- N_c-enhancement possible only for near by partons
  $\rightarrow$ only "color neighbors" radiate in the $N_c \rightarrow \infty$ limit

Backup: The size of the vector space and the trace basis

- For general N_c the trace type bases size grows as a factorial

  $$N_{\text{vec}}[n_q, N_g] = N_{\text{vec}}[n_q, N_g - 1](N_g - 1 + n_q) + N_{\text{vec}}[n_q, N_g - 2](N_g - 1)$$

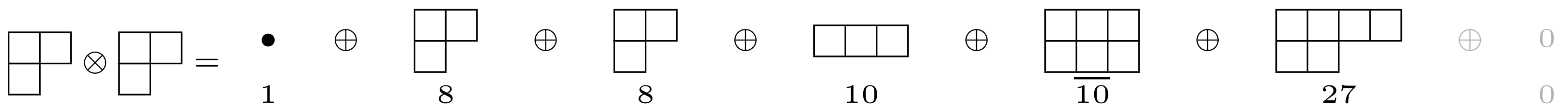
  where

  $$\begin{aligned} N_{\text{vec}}[n_q, 0] &= n_q! \\ N_{\text{vec}}[n_q, 1] &= n_q n_q! \end{aligned}$$

- The size of the vector spaces for finite N_c asymptotically grows as an exponential in the number of gluons/$q\bar{q}$-pairs.

Backup: Dealing with gluons

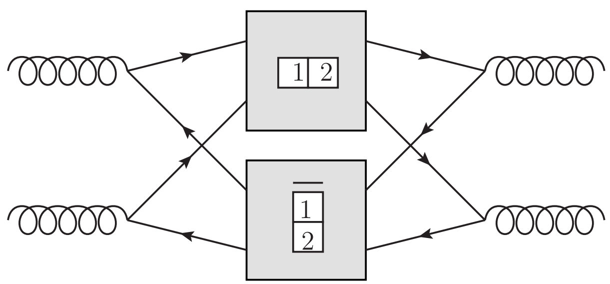
- Consider $gg \to gg$, the basis vectors can be enumerated using Young tableaux multiplication



- As color is conserved an incoming multiplet of a certain kind can only go to an outgoing multiplet of the same kind, $1 \to 1, 8 \to 8...$ $\to$ We know what to expect (Charge conjugation implies that some vectors only occur together)

- The problem is the construction of the corresponding projection operators; the Young tableaux operate with "quark-units" we need to deal also with gluons

- For two gluons, there are two octet projectors, one singlet projector, and 4 “new” projectors, 10, $\overline{10}, 27$, and for general N_c, “0”
- It turns out that the new projectors can be seen as corresponding to different symmetries w.r.t. quark and anti-quark units, for example the decuplet can be seen as corresponding to

$\mathbf{P}^{10} \propto$ 

$-$ octet(s) $-$ (singlet)

Similarly the anti-decuplet corresponds to $\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array} \otimes \overline{\begin{array}{|c|c|}\hline 1 & 2\\\hline\end{array}}$, the 27-plet corresponds to $\begin{array}{|c|c|}\hline 1 & 2\\\hline\end{array} \otimes \overline{\begin{array}{|c|c|}\hline 1 & 2\\\hline\end{array}}$ and the 0-plet to $\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array} \otimes \overline{\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}}$
(Dokshitzer and Marchesini 2006, others before using different methods)

Backup: New idea: Could this work in general?

On the one hand side

$$g_1 \otimes g_2 \otimes .... \otimes g_n \subseteq (q_1 \otimes \bar{\mathrm{q}}_1) \otimes (q_2 \otimes \bar{\mathrm{q}}_2) \otimes ... \otimes (q_n \otimes \bar{\mathrm{q}}_n)$$

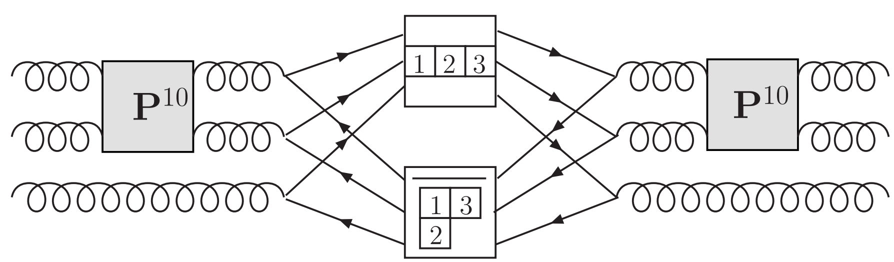
so there is hope...

On the other hand...

- Why should it?
- In general there are many instances of a multiplet, how do we know we construct all?

Key observation:

- Starting in a given multiplet, corresponding to some $q\overline{q}$ symmetries, such as 10, from $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline\end{array} \otimes \overline{\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline\end{array}}$, it turns out that for each way of attaching a quark box to $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline\end{array}$ and an anti-quark box to $\overline{\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline\end{array}}$, to there is at most one new multiplet! For example, the projector $\mathbf{P}^{10,35}$ can be seen as coming from



after having projected out "old" multiplets

- In fact, for large enough N_c, there is precisely one new multiplet for each set of $q\overline{q}$ symmetries

- In this way we can construct projection operators for an arbitrary number of gluons
- Using these we can find orthogonal minimal multiplet bases for any number of gluons
- From these we can construct *orthogonal minimal* bases for any number of quarks and gluons
- We have explicitly constructed orthogonal $3g \to 3g$ projectors and the corresponding six gluon orthogonal bases

JHEP09(2012)124, arXiv:1207.0609

Backup: Some example projectors

$$
\begin{aligned}
\mathbf{P}^{8a,8a}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R^2}\frac{1}{4N_c^2} if_{g_1\,g_2\,i_1} if_{i_1\,g_3\,i_2} if_{g_4\,g_5\,i_3} if_{i_3\,g_6\,i_2} \\
\mathbf{P}^{8s,27}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R}\frac{N_c}{2(N_c^2-4)} d_{g_1\,g_2\,i_1} \mathbf{P}^{27}_{i_1\,g_3\,i_2\,g_6} d_{i_2\,g_4\,g_5} \\
\mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{4(N_c+1)}{N_c^2(N_c+3)} \mathbf{P}^{27}_{g_1\,g_2\,i_1\,g_3} \mathbf{P}^{27}_{i_1\,g_6\,g_4\,g_5} \\
\mathbf{P}^{27,64=c111c111}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R^3} \mathbf{T}^{27,64}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} - \frac{N_c^2}{162(N_c+1)(N_c+2)} \mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} \\
&- \frac{N_c^2-N_c-2}{81N_c\,(N_c+2)} \mathbf{P}^{27,27s}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6}
\end{aligned}
$$

Backup: Three gluon multiplets

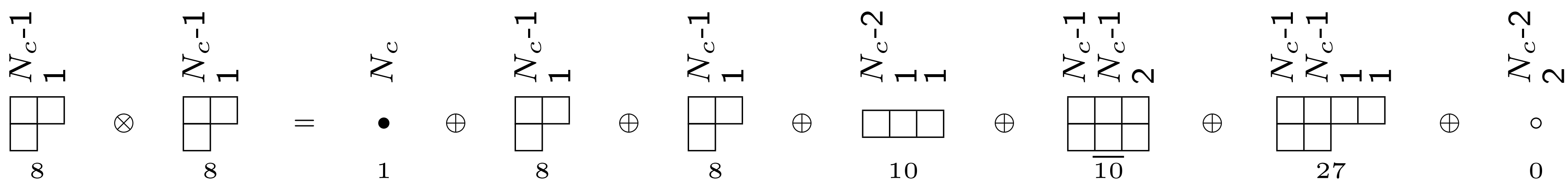
$SU(3)$ dim	1	8	10	$\overline{10}$	27	0
Multiplet	c0c0	c1c1	c11c2	c2c11	c11c11	c2c2
	$((45)^{8s}6)^{1}$	$2\times((45)^{8s}6)^{8s}$ or a	$((45)^{8s}6)^{10}$	$((45)^{8s}6)^{\overline{10}}$	$((45)^{8s}6)^{27}$	$((45)^{8s}6)^{0}$
	$((45)^{8a}6)^{1}$	$2\times((45)^{8a}6)^{8s}$ or a	$((45)^{8a}6)^{10}$	$((45)^{8a}6)^{\overline{10}}$	$((45)^{8a}6)^{27}$	$((45)^{8a}6)^{0}$
		$((45)^{10}6)^{8}$	$((45)^{10}6)^{10}$	$((45)^{\overline{10}}6)^{\overline{10}}$	$((45)^{10}6)^{27}$	$((45)^{10}6)^{0}$
		$((45)^{\overline{10}}6)^{8}$	$((45)^{10}6)^{10}$	$((45)^{\overline{10}}6)^{\overline{10}}$	$((45)^{\overline{10}}6)^{27}$	$((45)^{\overline{10}}6)^{0}$
		$((45)^{27}6)^{8}$	$((45)^{27}6)^{10}$	$((45)^{27}6)^{\overline{10}}$	$((45)^{27}6)^{27}$	$((45)^{0}6)^{0}$
		$((45)^{0}6)^{8}$	$((45)^{0}6)^{10}$	$((45)^{0}6)^{\overline{10}}$	$((45)^{27}6)^{27}$	$((45)^{0}6)^{0}$

$SU(3)$ dim	64	35	$\overline{35}$	0
Multiplet	c111c111	c111c21	c21c111	c21c21
	$((45)^{27}6)^{64}$	$((45)^{10}6)^{35}$	$((45)^{\overline{10}}6)^{\overline{35}}$	$((45)^{10}6)^{c21c21}$
		$((45)^{27}6)^{35}$	$((45)^{27}6)^{\overline{35}}$	$((45)^{\overline{10}}6)^{c21c21}$
				$((45)^{27}6)^{c21c21}$
				$((45)^{0}6)^{c21c21}$

$SU(3)$ dim	0	0	0	0	0
Multiplet	c111c3	c3c111	c21c3	c3c21	c3c3
	$((45)^{10}6)^{c111c3}$	$((45)^{\overline{10}}6)^{c3c111}$	$((45)^{10}6)^{c21c3}$	$((45)^{\overline{10}}6)^{c3c21}$	$((45)^{0}6)^{c3c3}$
			$((45)^{0}6)^{c21c3}$	$((45)^{0}6)^{c3c21}$	

Backup: SU(N_c) multiplets

- We find that the irreducible spaces in $g^{\otimes n_g}$ for varying N_c stand in a one to one, or one to zero correspondence to each other! (For each SU(3) multiplet there is an SU(5) version, but not vice versa)
- Every multiplet in $g^{\otimes n_g}$ can be labeled in an N_c-independent way using the lengths of the *columns*. For example



I have not seen this anywhere else.. have you?