LUND UNIVERSITY

A color matrix element corrected parton shower and multiplet color bases

- First results from an $SU(3)$, rather than $SU(\infty)$ parton shower

  In collaboration with Simon Plätzer (DESY), arXiv:1201.0260
- ColorFull – our treatment of the color structure
- Minimal orthogonal multiplet based color bases for treating the $SU(3)$ color space

  In collaboration with Stefan Keppeler (Tübingen), soon on the arXiv

DESY
June 1, 2012
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Why SU(3) parton showers?

- Wisdom from LEP is that parton showers seem to do well with the leading N_c approximation
- At LHC much more energy is available
  $\rightarrow$ many more colored partons
  $\rightarrow$ "many more squared" color suppressed terms
- Often two quark-lines $\rightarrow$ importance of terms suppressed by $1/N_c$ rather than $1/N_c^2$ should grow
- Also useful for exact NLO matching

Basics of our shower

- Built on the Catani-Seymour dipole factorization
  (S.Plätzer & S. Gieseke, JHEP 1101, 024 (2011) & 1109.6256)
- Parton $\tilde{ij}$ splitting to partons i and j, and parton $\tilde{k}$ absorbs the longitudinal recoil such that all partons remain on shell

$$
|\mathcal{M}_{n+1}(..., p_i, ..., p_j, ..., p_k, ...)|^2 \approx
\sum_{k \neq i,j} \frac{1}{2p_i \cdot p_j} \langle \mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)|\mathbf{V}_{ij,k}(p_i, p_j, p_k)|\mathcal{M}_n(..., p_{\tilde{ij}}, ..., p_{\tilde{k}}, ...)\rangle
$$

- In a standard parton shower parton $\tilde{ij}$ and $\tilde{k}$ would have to be "color connected",

$$
\mathbf{V}_{ij,k} = -8\pi\alpha_s V_{ij,k} \frac{\mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}}}{\mathbf{T}^2_{\tilde{ij}}} \to 8\pi\alpha_s \frac{V_{ij,k}}{1 + \delta_{\tilde{ij}}} \delta(\tilde{ij}, \tilde{k} \text{ color connected})
$$

we keep all pairs ($\delta_{\tilde{ij}} = 1$ for gluon, 0 else, $\mathbf{T}^2_{\tilde{ij}}$ is a convention)

For the emission probability this means that:

$$dP_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z) \frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times \frac{-1}{\mathbf{T}_{\tilde{ij}}^2} \frac{\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle}{|\mathcal{M}_n|^2}$$

rather than

$$dP_{ij,k}(p_\perp^2, z) = V_{ij,k}(p_\perp^2, z) \frac{d\phi_{n+1}(p_\perp^2, z)}{d\phi_n} \times \frac{\delta(\tilde{ij}, \tilde{k} \text{ color connected})}{1 + \delta_{\tilde{ij}}}$$

The splitting kernels read:

$$\begin{aligned} V_{qg,k}(p_i, p_j, p_k) &= C_F \left(\frac{2(1-z)}{(1-z)^2 + p_\perp^2/s_{ijk}} - (1+z) \right) \\ V_{gg,k}(p_i, p_j, p_k) &= 2C_A \left(\frac{1-z}{(1-z)^2 + p_\perp^2/s_{ijk}} + \frac{z}{z^2 + p_\perp^2/s_{ijk}} - 2 + z(1-z) \right) \end{aligned}$$

Challenges

Three major new challenges

- Evolution with amplitude information (next)
- Negative contributions to radiation probability, "negative splitting kernels", treated using interleaved veto/competition algorithm (S. Plätzer & M. Sjodahl, EPJ Plus 127 (2012) 26)
- Keeping track of the color structure for an arbitrary number of partons (more towards the end)

Evolution with amplitude information

- Assume we have a basis for the color space

$$|\mathcal{M}_n\rangle = \sum_{\alpha=1}^{d_n} c_{n,\alpha}|\alpha_n\rangle \quad \leftrightarrow \quad \mathcal{M}_n = (c_{n,1}, \ldots, c_{n,d_n})^T$$

  (this basis need not be orthogonal)

- $|\mathcal{M}_n\rangle$ is known for the hard process
- How do we get $|\mathcal{M}_{n+1}\rangle$ after emission?

- Observe that

$$|\mathcal{M}_n|^2 = \mathcal{M}_n^\dagger S_n \mathcal{M}_n = \mathrm{Tr}\left(S_n \times \mathcal{M}_n \mathcal{M}_n^\dagger\right)$$

(where S_n is the color scalar product matrix)
and

$$\langle \mathcal{M}_n | \mathbf{T}_{\tilde{ij}} \cdot \mathbf{T}_{\tilde{k}} | \mathcal{M}_n \rangle = \mathrm{Tr}\left(S_{n+1} \times T_{\tilde{k},n} \mathcal{M}_n \mathcal{M}_n^\dagger T^\dagger_{\tilde{ij},n}\right)$$

- Use an "amplitude matrix" $M_n = \mathcal{M}_n \mathcal{M}_n^\dagger$ as basic object

$$M_{n+1} = -\sum_{i \neq j} \sum_{k \neq i,j} \frac{4\pi\alpha_s}{p_i \cdot p_j} \frac{V_{ij,k}(p_i, p_j, p_k)}{\mathbf{T}^2_{\tilde{ij}}} \, T_{\tilde{k},n} M_n T^\dagger_{\tilde{ij},n}$$

where

$$M_{\mathsf{hard}} = \mathcal{M}_{\mathsf{hard}} \mathcal{M}^\dagger_{\mathsf{hard}}$$

Our current implementation

A proof of concept:

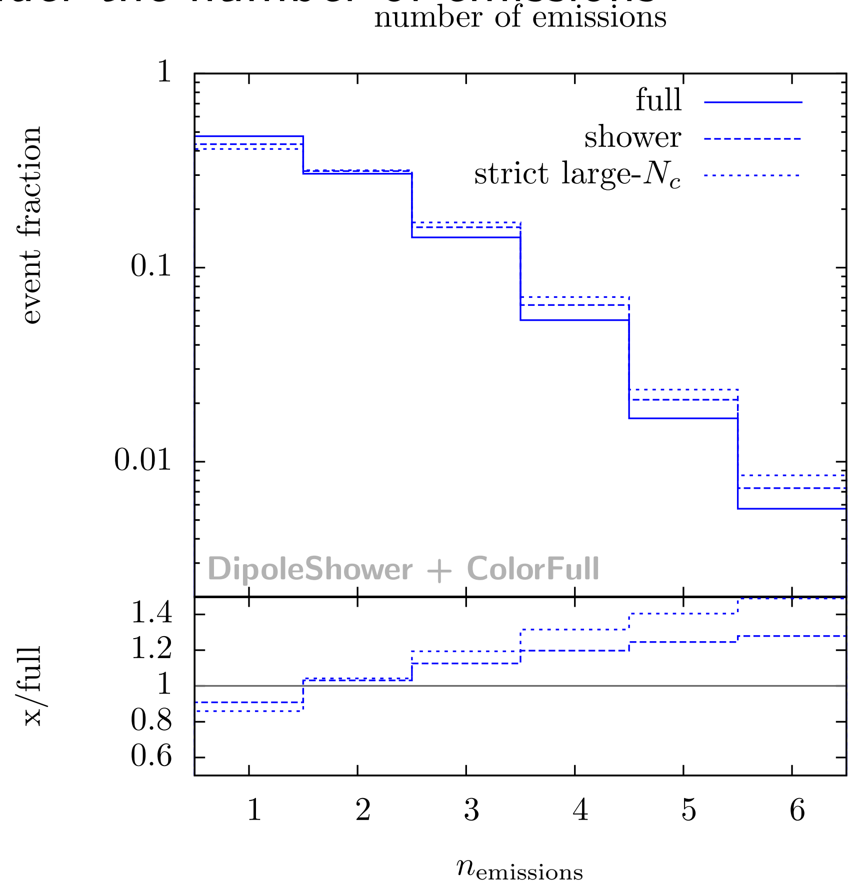
- $e^+e^- \longrightarrow$ jets, a LEP-like setting
- Fixed $\alpha_s = 0.112$
- Up to 6 gluons, only gluon emission, $g \longrightarrow q\bar{q}$ is suppressed anyway, and there is no non-trivial color structure
- No hadronization, we don't want to spoil our $N_c = 3$ parton shower by attaching an $N_c \to \infty$ hadronization model. Also, comparing showers in a fair way, would require retuning the hadronized $N_c = 3$ shower
- No "virtual" corrections, i.e. no color rearrangement without radiation, no Coulomb gluons

Three different treatments of color space

- Full, exact SU(3) treatment, all color correlations
- Shower, resembles standard showers, C_F for gluon emission off quarks is exact but non-trivial color suppressed terms are dropped
- Strict large-N_c, all N_c suppressed terms dropped, $C_F = 4/3 \rightarrow 3/2$ $(T_R = 1/2)$

Results: Number of emissions

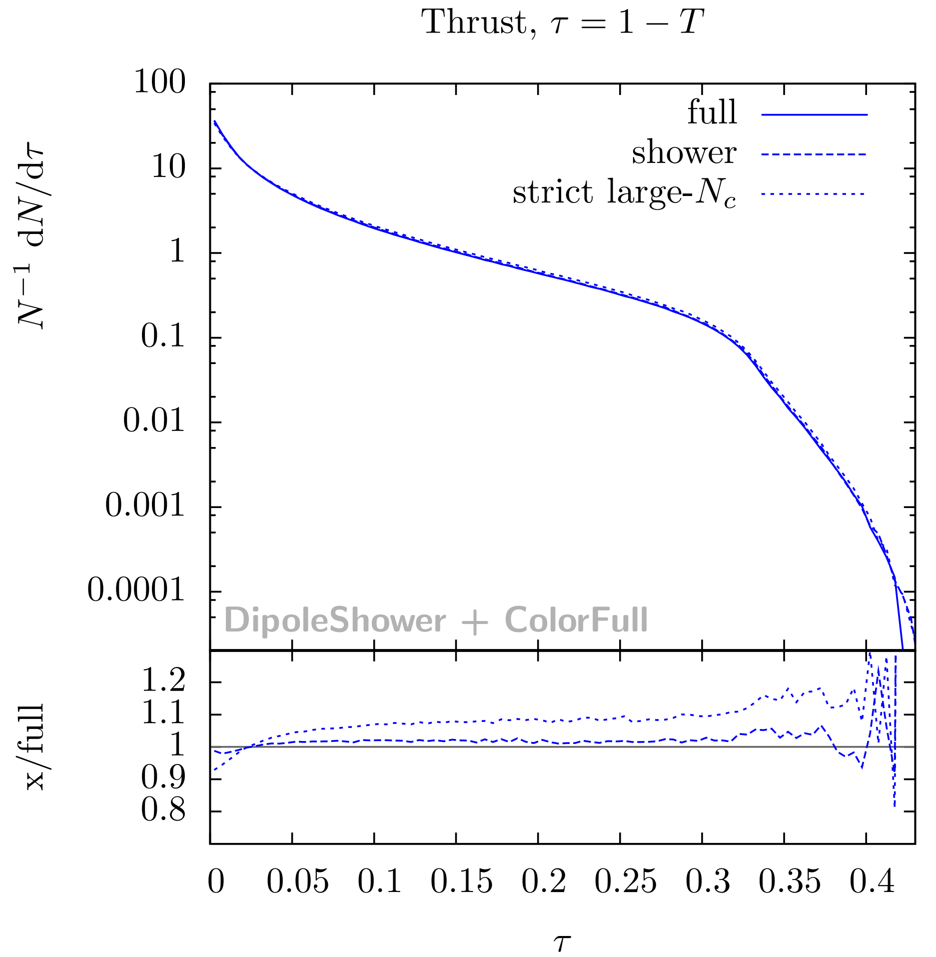
First, simply consider the number of emissions



... this is not an observable, but it is a genuine uncertainty on the number of emissions in the perturbative part of a parton shower

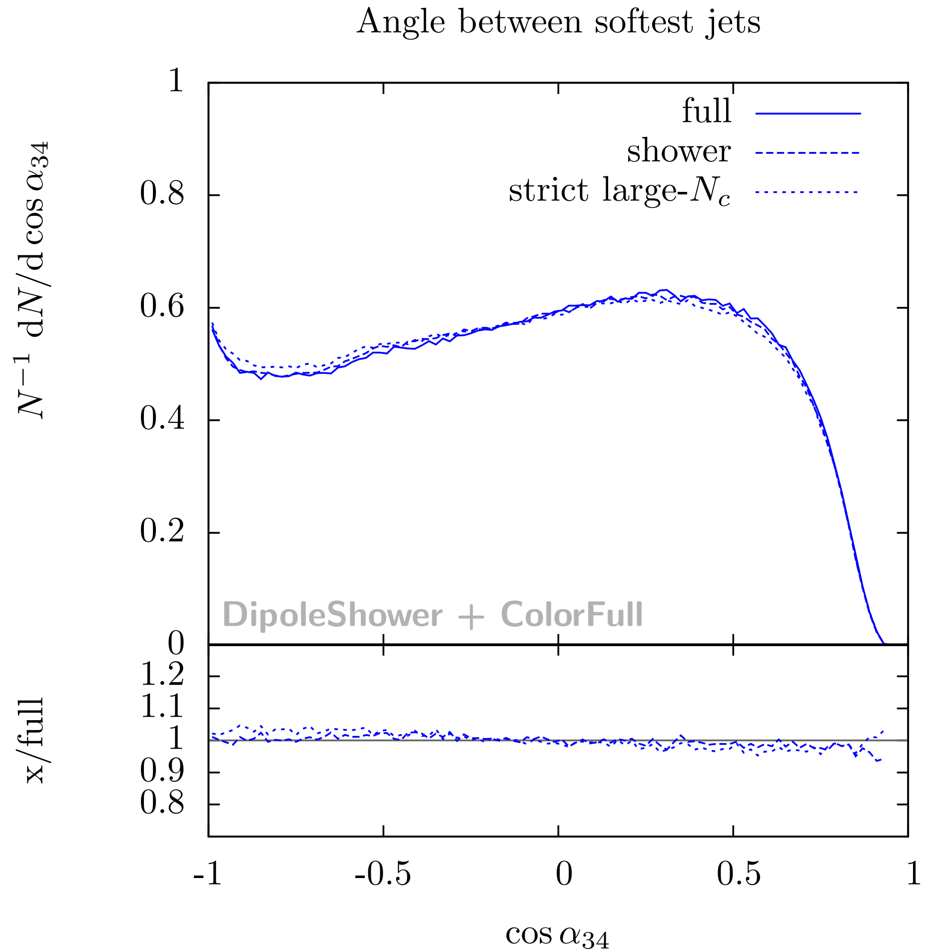
Results: Thrust

For standard observables small effects, here thrust $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$



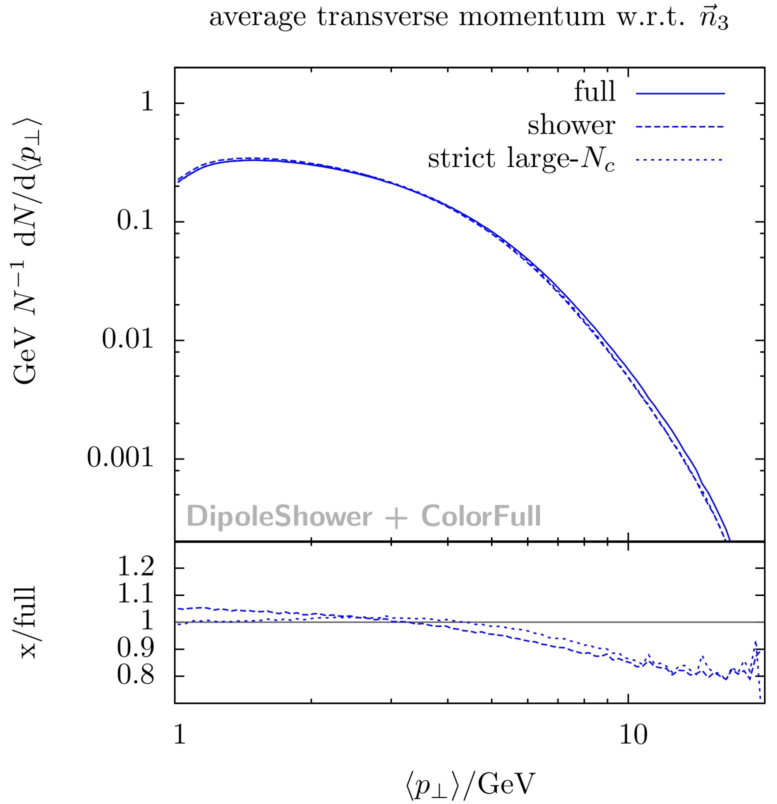
Results: Angular distribution

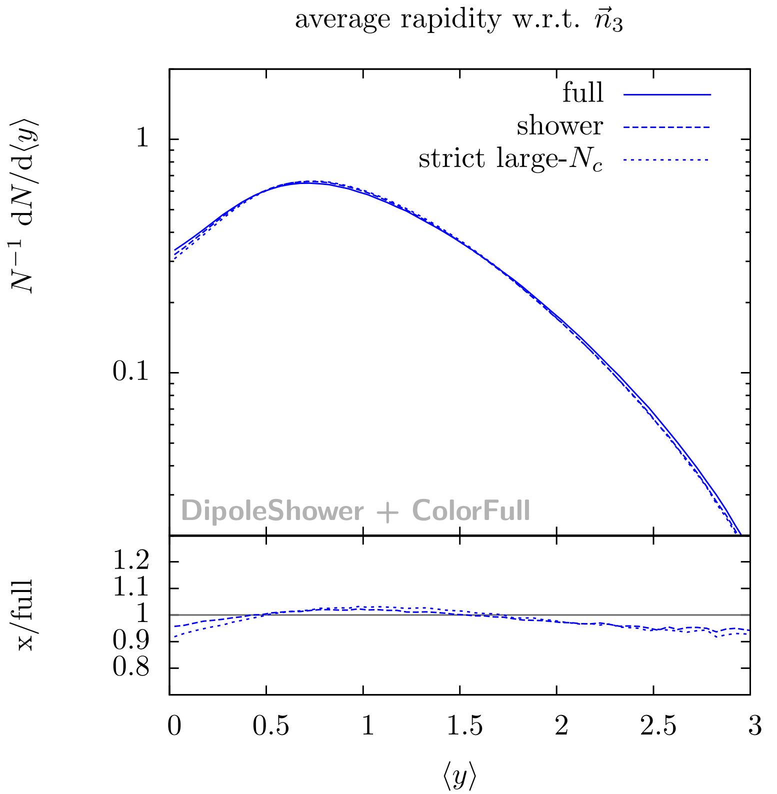
Cosine of angle between third and fourth jet



Results: Some tailored observables

For tailored observables we find larger differences





Average transverse momentum and rapidity of softer particles with respect to the thrust axis defined by the three hardest partons

Parton shower conclusions

- For standard observables we find small deviations for LEP, of order a few percent
- Leading N_c was probably a very good approximation for standard observables at LEP
- For tailored observables we find larger differences $\approx 20\%$
- Keeping C_F to its $N_c = 3$ value $(4/3)$ (as is done in standard showers), rather than $3/2$, tends to improve the approximation $(T_R = 1/2)$
- At the LHC we have many more colored particles, so (many more)^2 possible color suppressed interference terms
- For full evolution we should include color rearranging virtual corrections, they do have the same IR singularity structure

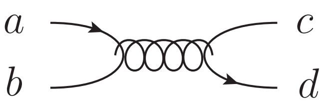
The color space

- For given external particles, the color space is a finite dimensional vector space equipped with a scalar product

$$\langle A, B\rangle = \sum_{a,b,c,\ldots} A_{a,b,c,\ldots}(B_{a,b,c,\ldots})^*$$

Example: If

$$A = (t^g)^a{}_b(t^g)^c{}_d =$$



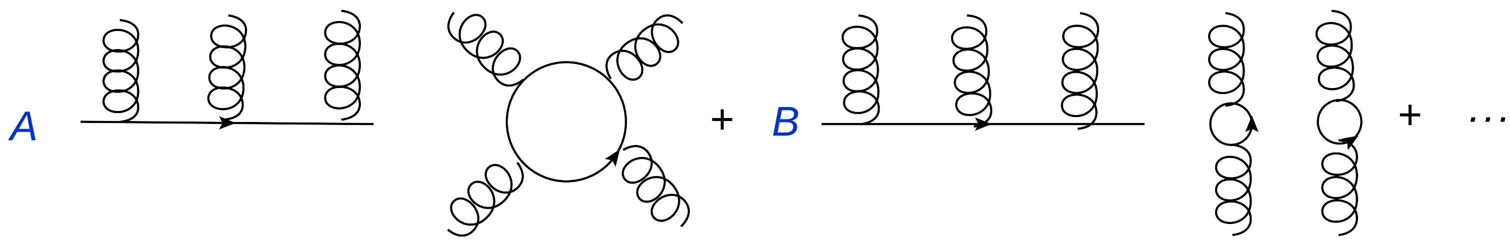
,

then $\langle A|A\rangle = \sum_{a,b,c,d,g,f}(t^g)^a{}_b(t^g)^c{}_d(t^h)^b{}_a(t^h)^d{}_c$

- We may use any basis

A basis for the color space

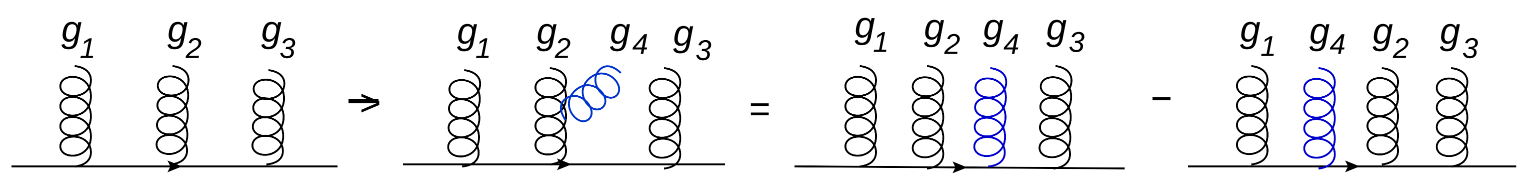
- In general an amplitude can be written as linear combination of different color structures, like



- This is the kind of "trace" bases used in our current parton shower, and most NLO calculations

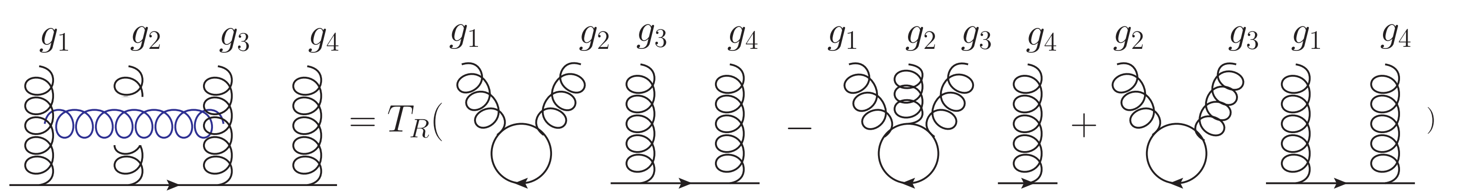
It has some nice properties

- The effect of gluon emission is easily described:



(Z. Nagy & D. Soper, JHEP 0807 (2008) 025)

- So is the effect of gluon exchange:



Convention: + when inserting after, - when inserting before

(M. Sjödahl, JHEP 0909 (2009) 087 JHEP)

ColorFull

For the purpose of treating a general QCD color structure I have written a C++ color algebra code, ColorFull, which:

- automatically creates a "trace" basis for any number and kind of partons, and to any order in α_{s}
- describes the effect of gluon emission
- ... and gluon exchange
- squares color amplitudes
- can be used with boost for optimized calculations
- is planned to be published separately

But...

- this type of basis is non-orthogonal and overcomplete
  (for more than a few partons)
- ... and the number of basis vectors grows as a factorial in N_g
  $\rightarrow$ when squaring amplitudes we run into a factorial square scaling
- Hard to go beyond $q\overline{q}+7$ gluons

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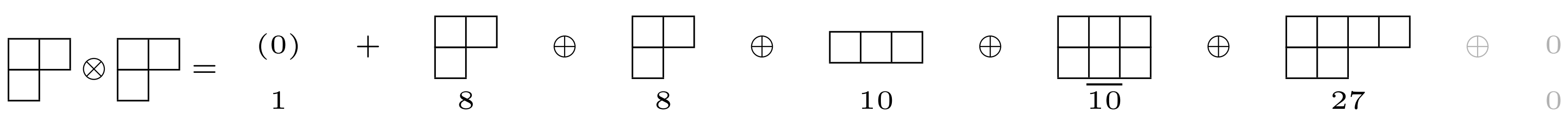
  (for more than a few partons)
- ... and the number of basis vectors grows as a factorial in N_g
  $\to$ when squaring amplitudes we run into a factorial square scaling
- Hard to go beyond $q\overline{q} + 7$ gluons
- Would be nice with minimal orthogonal basis

Minimal orthogonal bases for the color spaces

In collaboration with Stefan Keppeler

- Want orthogonal minimal basis for color space
- Basis vectors can be enumerated using Young tableaux multiplication



and constructed if projection operators are known

- The problem is the construction of the corresponding projection operators; the Young-tableaux operate with ”quark-units” but the physical particles include anti-quarks and gluons

- New idea: Iteratively build up gluon projection operators using quark and anti-quark projection operators and project out already known projection operators
- We have found a general strategy for constructing gluon projection operators for $n_g \to n_g$ gluons!
- This can be used for constructing orthogonal bases for up to $2n_g + 1$ gluons or $q\bar{q}$-pairs!
- We have explicitly constructed the 51 $3g \to 3g$ projection operators for any N_c
- ... and the 6 gluon orthonormal bases
- ... and orthonormal bases for all other 6 parton cases
- Bases can easily be made minimal by crossing out states that are disallowed for $N_c = 3$

Number of projection operators and basis vectors

Number of projection operators and basis vectors for $N_g \to N_g$ gluons *without* imposing projection operators and vectors to appear in charge conjugation invariant combinations

Case	Projectors $N_c = 3$	Projectors $N_c = \infty$	Vectors $N_c = 3$	Vectors $N_c = \infty$
2g $\to$ 2g	6	7	8	9
3g $\to$ 3g	29	51	145	265
4g $\to$ 4g	166	513	3 598	14 833
5g $\to$ 5g	1 002	6 345	107 160	1 334 961

Conclusions for the color space

- We have outlined a general recipe for construction of minimal orthogonal multiplet based bases for any QCD process
- On the way we found an N_c-independent labeling of the multiplets in $g^{\otimes n_g}$, and a one to one, or one to zero, correspondence between these for various N_c
- ... and an N_c independent way of obtaining $SU(N_c)$ Clebsch-Gordan matrices
- Number of basis vectors grows only exponentially for $N_c = 3$
- This has the potential to very significantly speed up exact calculations in the color space of $SU(N_c)$

...and outlook

- However, in order to use this in an optimized way, we need to understand how to sort QCD amplitudes in this basis in an efficient way
- ...also, a lot of implementational work remains

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- ...also, a lot of implementational work remains

Thank you for your attention

Backup: The Sudakov decomposition

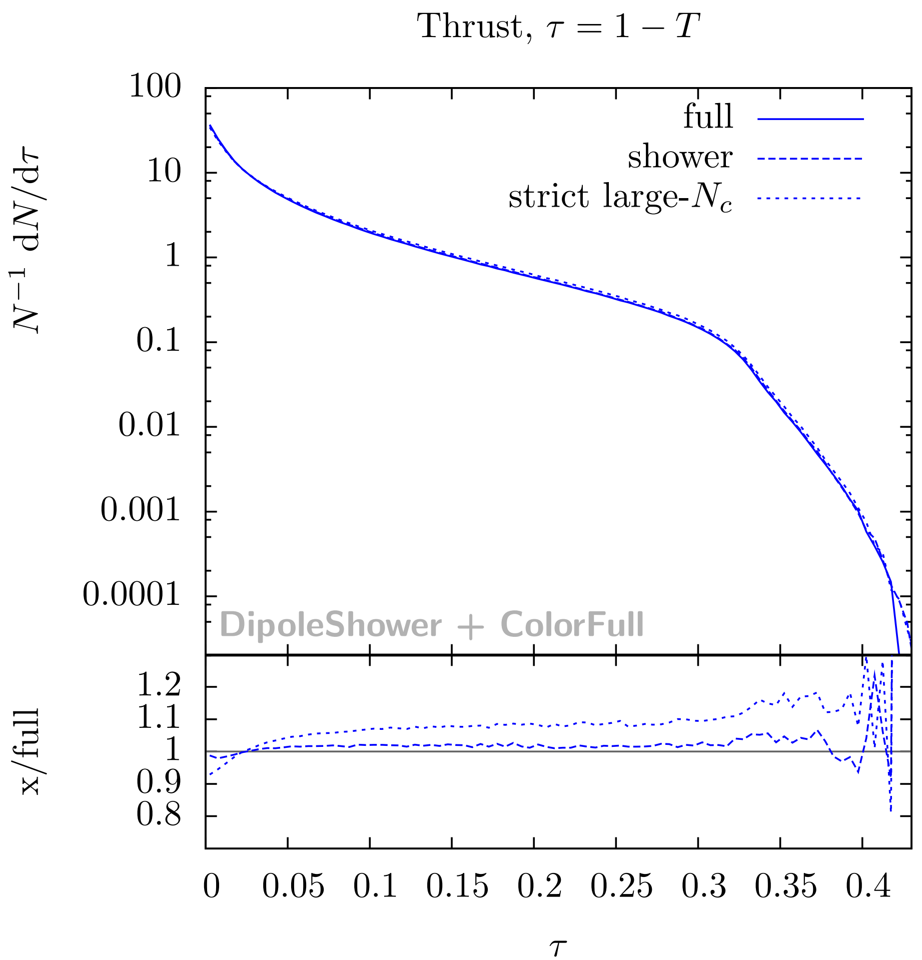
In each splitting a parton $\tilde{ij}$ splits into i and j whereas a spectator $\tilde{k}$ takes up the longitudinal recoil

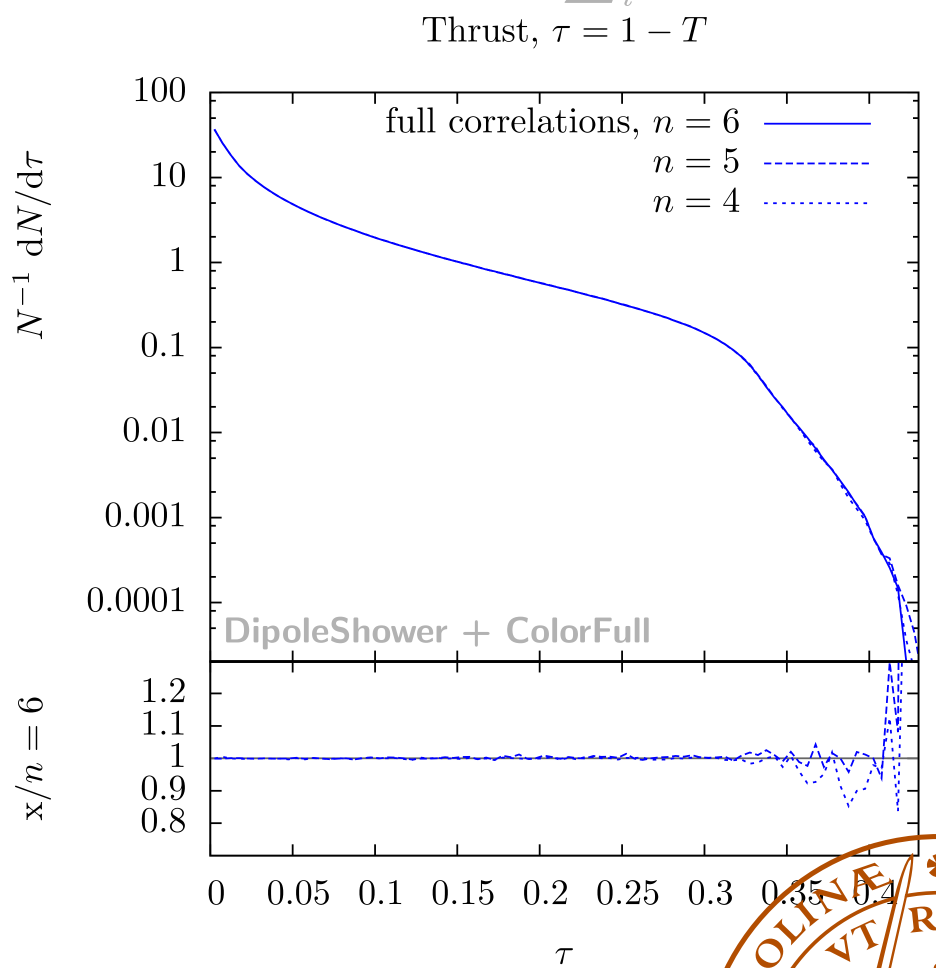
$$\begin{aligned} p_i &= z p_{\tilde{ij}} + \frac{p_\perp^2}{z s_{ijk}} p_{\tilde{k}} + k_\perp & (1) \\ p_j &= (1-z) p_{\tilde{ij}} + \frac{p_\perp^2}{(1-z) s_{ijk}} p_{\tilde{k}} - k_\perp & (2) \\ p_k &= \left(1 - \frac{p_\perp^2}{z(1-z) s_{ijk}}\right) p_{\tilde{k}} \, , & (3) \end{aligned}$$

with $p_{\tilde{ij}}^2 = p_{\tilde{k}}^2 = 0$, a space like transverse momentum $k_\perp$ with $k_\perp^2 = -p_\perp^2$ and $k_\perp \cdot p_{\tilde{ij}} = k_\perp \cdot p_{\tilde{k}} = 0$. With this parametrization we also have $s_{ijk} = (p_i + p_j + p_k)^2 = (p_{\tilde{ij}} + p_{\tilde{k}})^2$.

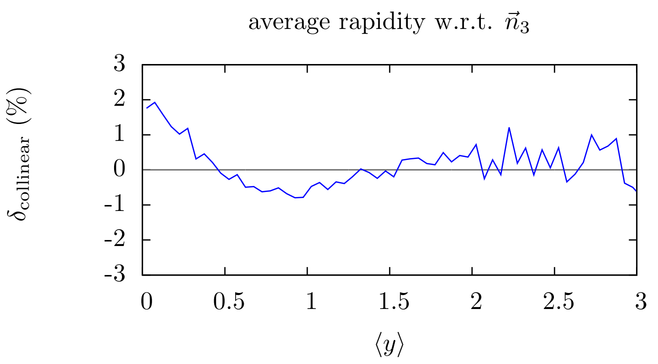
Backup: Thrust

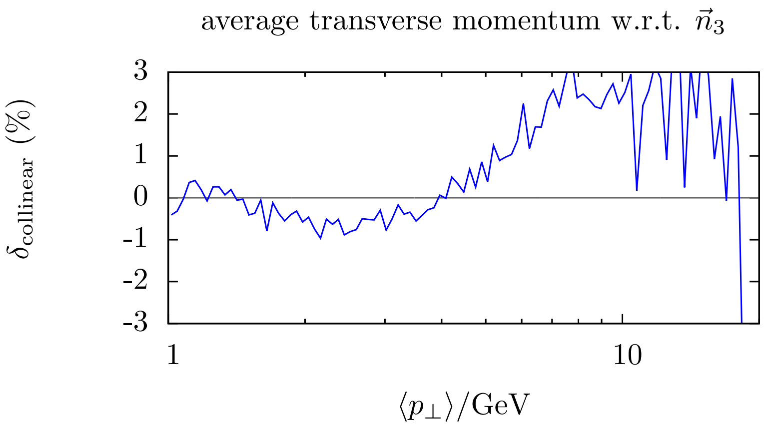
For standard observables small effects, here thrust $T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|}$





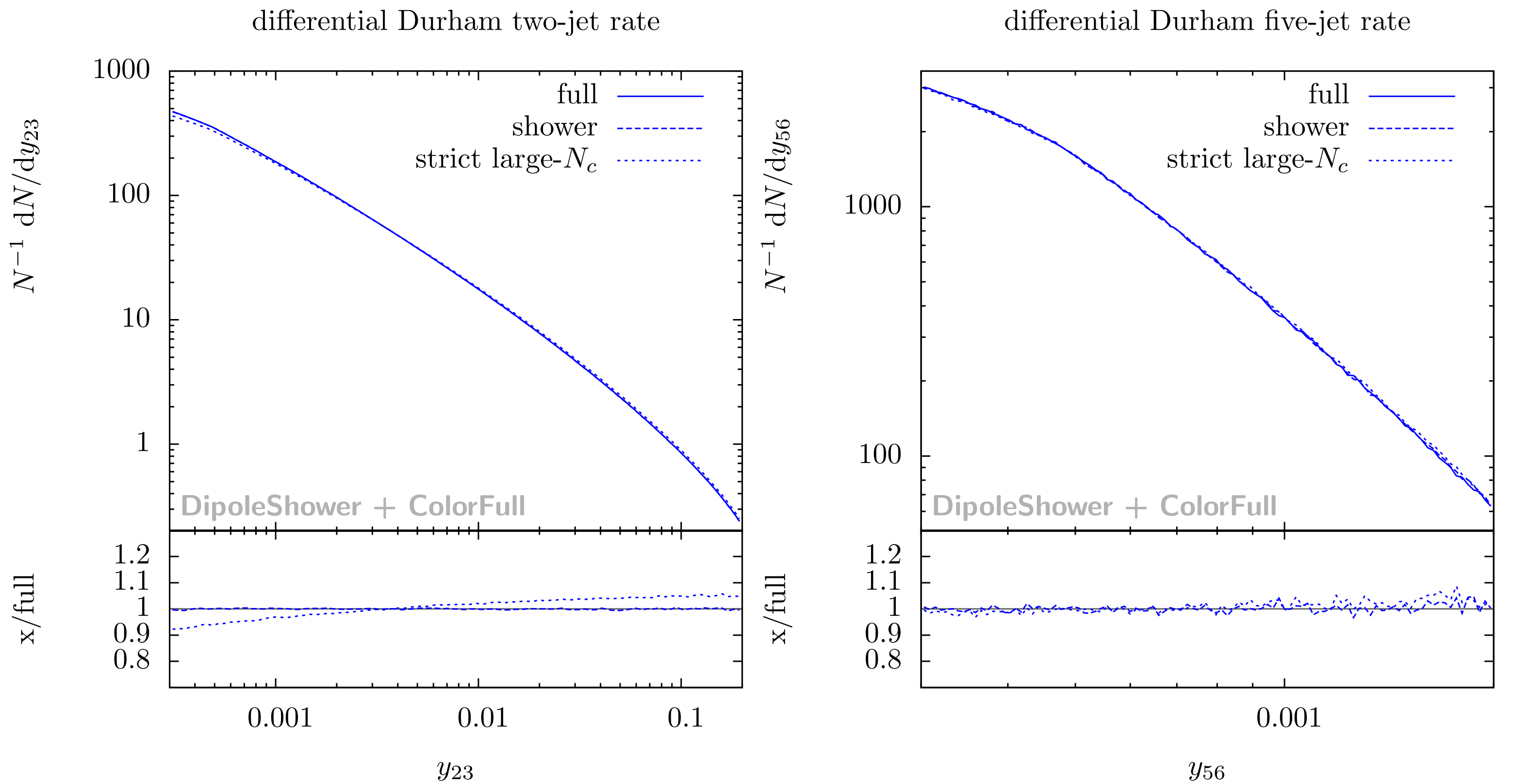
Backup: Importance of $g \to q\bar{q}$ splitting





Influence on average transverse momentum and rapidity w.r.t. the thrust axis defined by the three hardest patrons

Backup: Jet separation

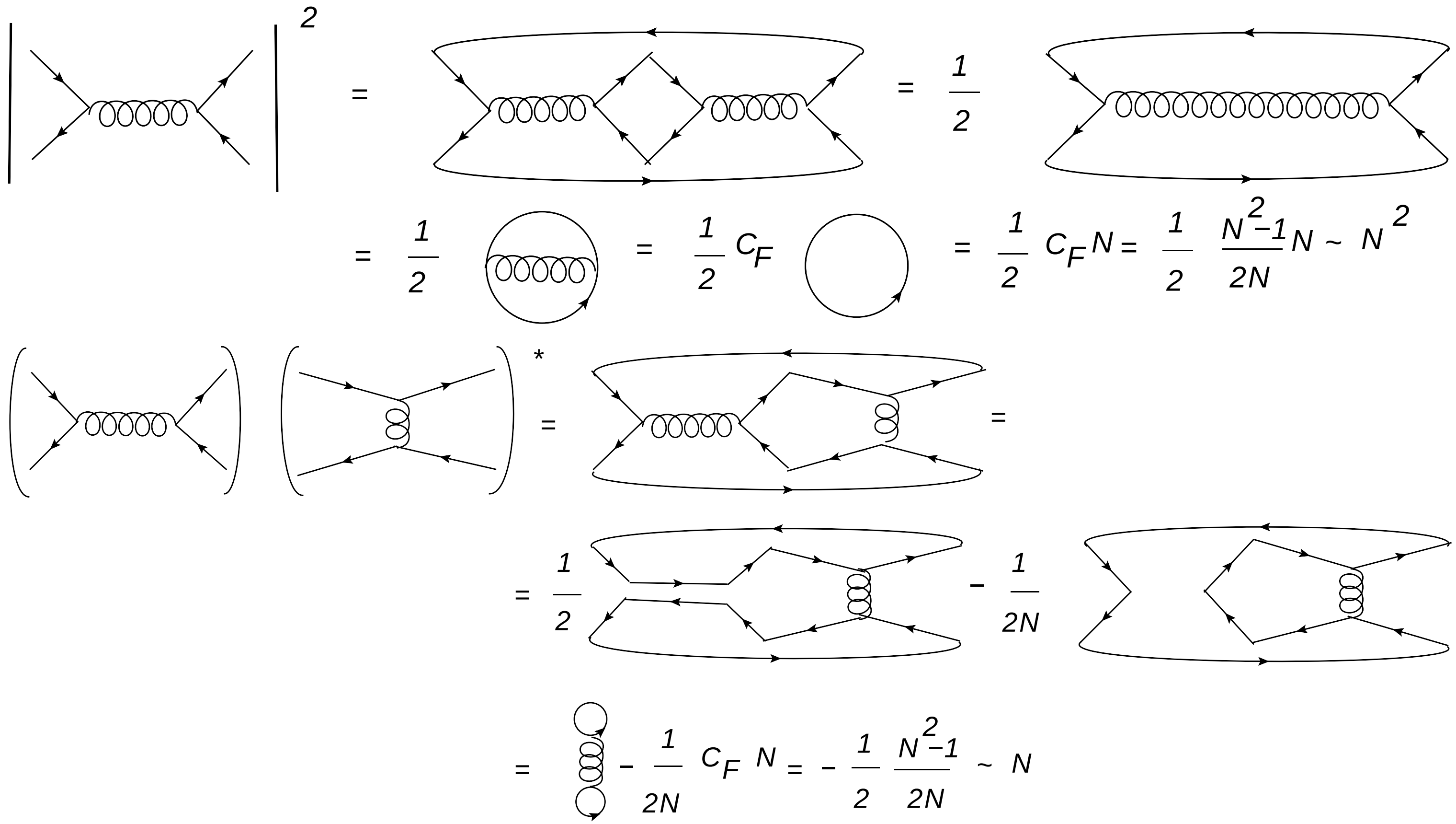


Jet separation between 2nd and 3rd jet, and 5th and 6th jet

$y = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})/s$

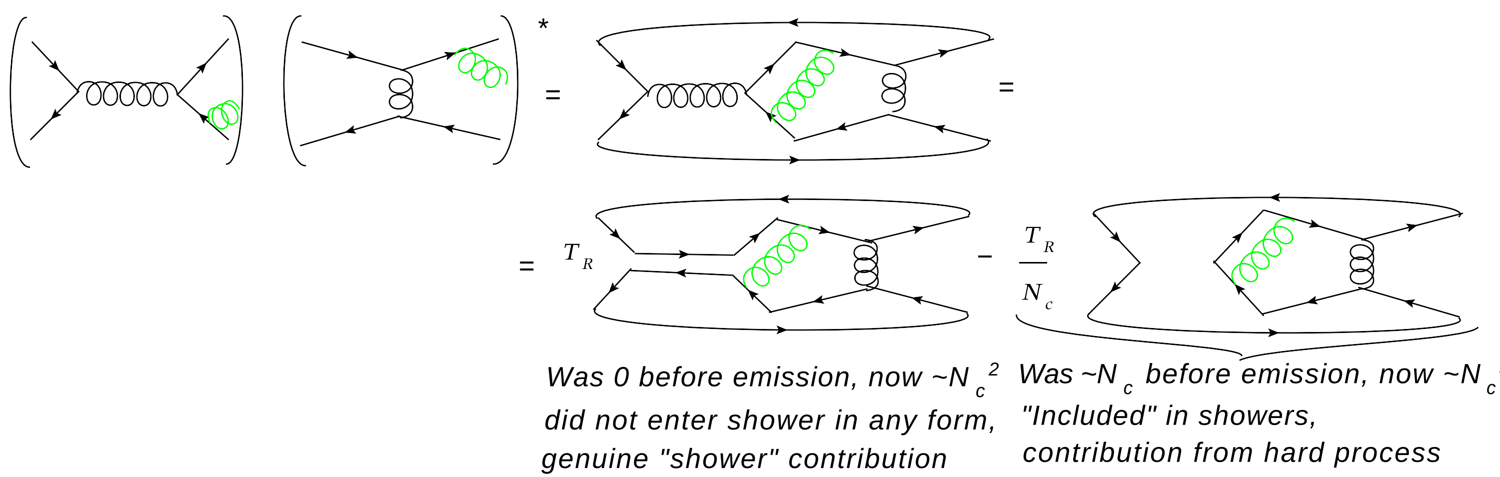
Backup: N_c suppressed terms

That non-leading color terms are suppressed by $1/N_c^2$, is guaranteed only for same order α_s diagrams with only gluons ('t Hooft 1973)



Backup: N_c suppressed terms

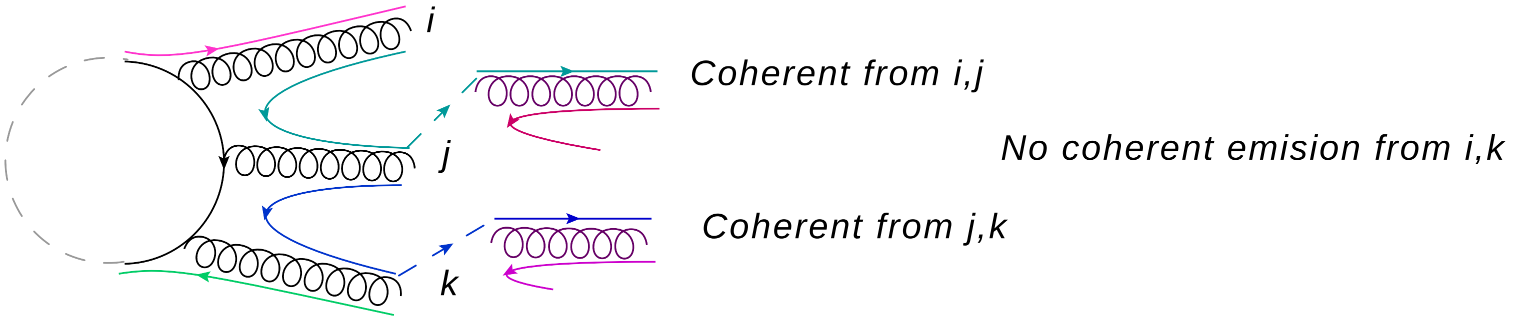
For a parton shower there may also be terms which only are suppressed by one power of N_c



The leading N_c contribution scales as N_c^2 before emission and N_c^3 after

Backup: A dipole shower in the "trace" basis

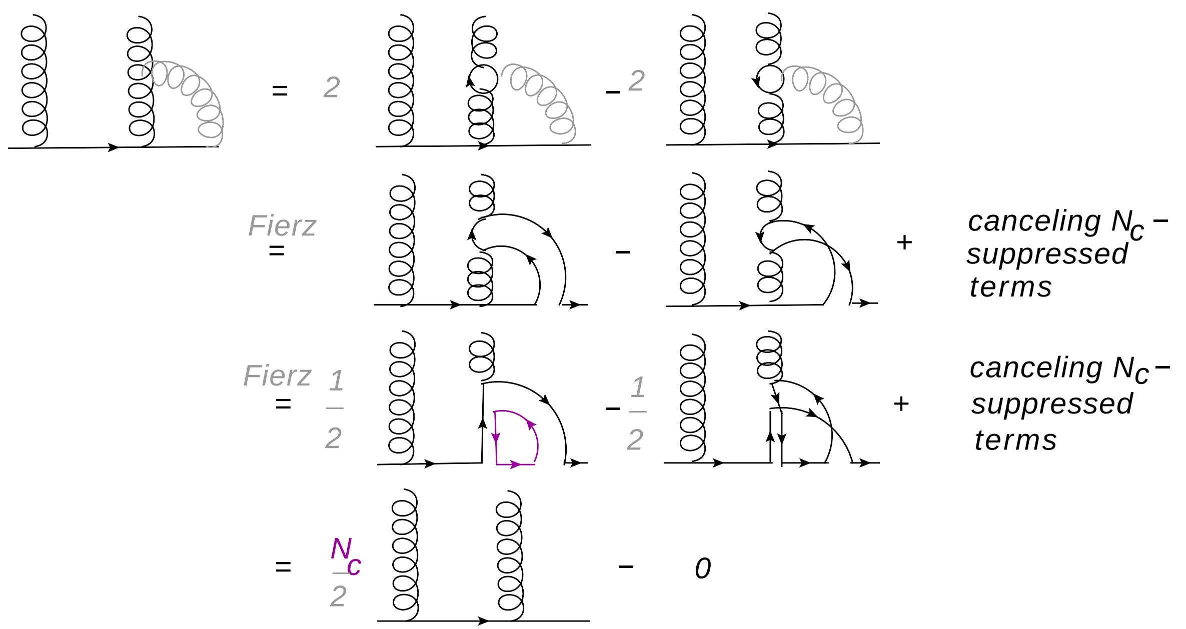
- A dipole shower can easily be thought of in the language of the $N_c \to \infty$ limit of the "trace" basis



- Also, it is easy to see that in this limit only "color neighbors" radiate, i.e. only neighboring partons on the quark-lines in the basis radiate $\rightarrow$ trace basis well suited for comparing to parton showers

Backup: Gluon exchange

A gluon exchange in this basis "directly" i.e. without using scalar products gives back a linear combination of (at most 4) basis tensors



- N_c-enhancement possible only for near by partons
  $\longrightarrow$ only "color neighbors" radiate in the $N_c \to \infty$ limit

Backup: The size of the vector space and the trace basis

- For general N_c the trace type bases size grows as a factorial

  $$N_{\text{vec}}[n_q, N_g] = N_{\text{vec}}[n_q, N_g - 1](N_g - 1 + n_q) + N_{\text{vec}}[n_q, N_g - 2](N_g - 1)$$

  where

  $$\begin{aligned} N_{\text{vec}}[n_q, 0] &= n_q! \\ N_{\text{vec}}[n_q, 1] &= n_q n_q! \end{aligned}$$

- The size of the vector spaces for finite N_c asymptotically grows as an exponential in the number of gluons/$q\overline{q}$-pairs.

Backup: Some example projectors

$$
\begin{aligned}
\mathbf{P}^{8a,8a}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R^2}\frac{1}{4N_c^2} if_{g_1\,g_2\,i_1} if_{i_1\,g_3\,i_2} if_{g_4\,g_5\,i_3} if_{i_3\,g_6\,i_2} \\
\mathbf{P}^{8s,27}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R}\frac{N_c}{2(N_c^2-4)} d_{g_1\,g_2\,i_1}\mathbf{P}^{27}_{i_1\,g_3\,i_2\,g_6} d_{i_2\,g_4\,g_5} \\
\mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{4(N_c+1)}{N_c^2(N_c+3)}\mathbf{P}^{27}_{g_1\,g_2\,i_1\,g_3}\mathbf{P}^{27}_{i_1\,g_6\,g_4\,g_5} \\
\mathbf{P}^{27,64=c111c111}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} &= \frac{1}{T_R^3}\mathbf{T}^{27,64}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} - \frac{N_c^2}{162(N_c+1)(N_c+2)}\mathbf{P}^{27,8}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6} \\
&- \frac{N_c^2-N_c-2}{81N_c\,(N_c+2)}\mathbf{P}^{27,27s}_{g_1\,g_2\,g_3\,g_4\,g_5\,g_6}
\end{aligned}
$$

Backup: Three gluon multiplets

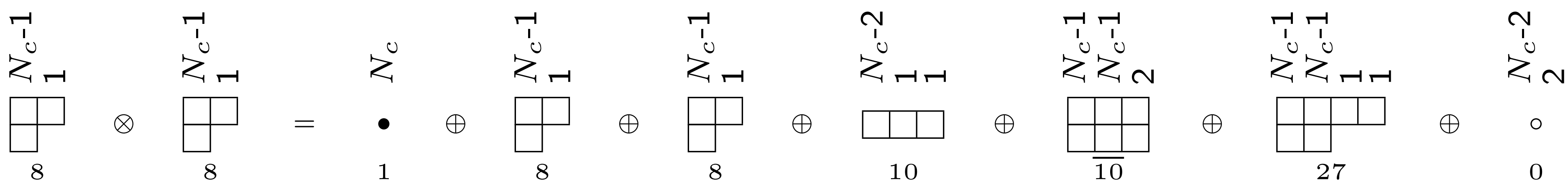
$SU(3)$ dim	1	8	10	$\overline{10}$	27	0
Multiplet	c0c0	c1c1	c11c2	c2c11	c11c11	c2c2
	$((45)^{8s}6)^{1}$	$2\times((45)^{8s}6)^{8s \text{ or } a}$	$((45)^{8s}6)^{10}$	$((45)^{8s}6)^{\overline{10}}$	$((45)^{8s}6)^{27}$	$((45)^{8s}6)^{0}$
	$((45)^{8a}6)^{1}$	$2\times((45)^{8a}6)^{8s \text{ or } a}$	$((45)^{8a}6)^{10}$	$((45)^{8a}6)^{\overline{10}}$	$((45)^{8a}6)^{27}$	$((45)^{8a}6)^{0}$
		$((45)^{10}6)^{8}$	$((45)^{10}6)^{10}$	$((45)^{\overline{10}}6)^{\overline{10}}$	$((45)^{10}6)^{27}$	$((45)^{10}6)^{0}$
		$((45)^{\overline{10}}6)^{8}$	$((45)^{10}6)^{10}$	$((45)^{\overline{10}}6)^{\overline{10}}$	$((45)^{\overline{10}}6)^{27}$	$((45)^{\overline{10}}6)^{0}$
		$((45)^{27}6)^{8}$	$((45)^{27}6)^{10}$	$((45)^{27}6)^{\overline{10}}$	$((45)^{27}6)^{27}$	$((45)^{0}6)^{0}$
		$((45)^{0}6)^{8}$	$((45)^{0}6)^{10}$	$((45)^{0}6)^{\overline{10}}$	$((45)^{27}6)^{27}$	$((45)^{0}6)^{0}$

$SU(3)$ dim	64	35	$\overline{35}$	0
Multiplet	c111c111	c111c21	c21c111	c21c21
	$((45)^{27}6)^{64}$	$((45)^{10}6)^{35}$	$((45)^{\overline{10}}6)^{\overline{35}}$	$((45)^{10}6)^{c21c21}$
		$((45)^{27}6)^{35}$	$((45)^{27}6)^{\overline{35}}$	$((45)^{\overline{10}}6)^{c21c21}$
				$((45)^{27}6)^{c21c21}$
				$((45)^{0}6)^{c21c21}$

$SU(3)$ dim	0	0	0	0	0
Multiplet	c111c3	c3c111	c21c3	c3c21	c3c3
	$((45)^{10}6)^{c111c3}$	$((45)^{\overline{10}}6)^{c3c111}$	$((45)^{10}6)^{c21c3}$	$((45)^{\overline{10}}6)^{c3c21}$	$((45)^{0}6)^{c3c3}$
			$((45)^{0}6)^{c21c3}$	$((45)^{0}6)^{c3c21}$	

Backup: SU(N_c) multiplets

- We find that the irreducible spaces in $g^{\otimes n_g}$ for varying N_c stand in a one to one, or one to zero correspondence to each other! (For each SU(3) multiplet there is an SU(5) version, but not vice versa)
- Every multiplet in $g^{\otimes n_g}$ can be labeled in an N_c-independent way using the lengths of the *columns*. For example



I have not seen this anywhere else.. have you?