Problems for problem session May 8, 2015

(1.) Consider a particle with mass m and charge q moving with the velocity $\vec{v}$ in the electromagnetic field from a plane wave moving in the z-direction. The fields are determined by Maxwells eqn's in vacuum and the four-vector potential $A^\mu = \left(\frac{1}{c}V, \vec{A}\right)$ is given by

$$A^\mu = \left(0,\ \hat{e}\,\tilde{A}\,e^{i(\tilde{k}\cdot\vec{r}-\omega t)}\right)$$

where $\hat{e}$ is a unit vector and $\tilde{A}$ is a real amplitude

a) determine the $\vec{E}$- and $\vec{B}$-fields

b) calculate the force on the particle

c) specialize to the case $\vec{v} = v\hat{e}$. How large is the magnetic force and in what direction?

(cf. problem 9.11)

(2.) Show that Maxwell's equations in vacuum without any charges or currents are invariant under the transformation

$$\vec{E}' = a\vec{E} + bc\vec{B}$$

$$\vec{B}' = -\frac{b}{c}\vec{E} + a\vec{B}$$

where a, b are constants and c is the speed of light

b) How should a and b be related in order that the energy density is invariant?

(cf. Problem 7.64)