Additional problems May 22, 2015

1. Consider a plane EM wave with angular frequency ω impinging from vacuum on a plane surface of an excellent conductor ($\mu = \mu_0$, $\sigma \gg \omega\varepsilon$) with normal incidence.

   Assume the incoming wave is given by

$$\tilde{\vec{E}}_i = \hat{x}\, E_i\, e^{i(kz-\omega t)}$$

a) calculate the skin depth δ

b) calculate the refractive index $\tilde{n}_2$ in the conductor and show that $\tilde{n}_2 = (1+i)\frac{c}{\delta\omega}$

c) calculate the $\vec{E}$-field of the transmitted wave

d) what is the current in the conductor expressed in the skin depth?

e) determine the reflected wave in the case of a perfect conductor, $\sigma \to \infty$

(2) Consider two plane waves moving in the z-direction with electric field

$$\tilde{\vec{E}} = \hat{e}\, E\, e^{i((k+\Delta k)z - (\omega+\Delta\omega)t)} + \hat{e}\, E\, e^{i((k-\Delta k)z - (\omega-\Delta\omega)t)}$$

where E is real and $\Delta k \ll k$, $\Delta\omega \ll \omega$

a) Show that the physical field is given by

$$\vec{E} = 2\hat{e}\, E \cos(\Delta k \cdot z - \Delta\omega \cdot t) \cos(kz - \omega t)$$

b) What is the wavelength for the carrier wave which is described by $\cos(kz - \omega t)$, and for the modulation described by $\cos(\Delta k \cdot z - \Delta\omega \cdot t)$? What is the ratio?

c) Sketch the solution in a) at the time $t = 0$ and at a somewhat later time. Identify the phase and group velocities in your sketch.

3.) Consider an electric dipole in the z-direction with harmonic time-dependence (angular frequency ω) placed in the origin

In the radiation zone ($kr \gg 1$) the electric field is given by

$$\tilde{\vec{E}}(\vec{r},t) = \frac{e^{i(kr-\omega t)}}{r} E_0 \sin\theta \, \hat{\theta}$$

a) Calculate the flux through a quadratic planar loop placed on the x-axis a distance L from the dipole with $L \gg a$, $a \ll \lambda = \frac{2\pi}{k}$

b) What is the maximal amplitude of the current induced in the loop if it has a resistance R.

c) Discuss qualitatively what EM-fields the current in the loop will give rise to.