Practice questions — chapters 7, 8.6.2–3 and 9

1. Osmotic pressure. Derive and explain the van't Hoff relation $p_{\text{equil}} = ck_{\text{B}}T$.

2. Depletion interaction. Consider the system illustrated by figure 7.3 in Nelson's book. Assuming that the concentration of small particles, c, is low, show that the free-energy difference $\Delta F \equiv F(l) - F(2R)$ is given by $\Delta F = -ck_{\text{B}}T(2R - l)A$ for $l < 2R$.

3. What is the meaning of the Bjerrum length, and what is its value in air and water, respectively?

4. Consider a solution of ions with charge e in water. Derive the Poisson-Boltzmann equation for the potential V, eq. 7.23 in Nelson's book (where $\bar{V} = eV/k_{\text{B}}T$). What is the relation between V and the ion concentration c_+? The book calculates the ion distribution in the presence of a negatively charged surface. How does the surface enter into the calculation?

5. Figure 7.14 in Nelson's book shows solubilities of small nonpolar molecules in water. The measured solubilities decrease with increasing temperature. How can this behavior be explained?

6. Hydrophobic interactions are a major driving force in protein folding. Describe three observations supporting this conclusion.

7. Figure 9.4 i Nelson's book shows force versus extension in a single-molecule stretching experiment on DNA. Describe the different regimes seen (A-E).

8. Consider the elastic rod (wormlike chain) model, with energy

$$E = \frac{Ak_{\text{B}}T}{2} \int_0^{L_{\text{tot}}} \boldsymbol{\beta}^2 \mathrm{d}s$$

   Suppose the rod is bent into a quarter-circle of radius R. Show that $E = \pi Ak_{\text{B}}T/4R$. This result indicates that whether or not the rod will be bent by thermal fluctuations depends on the relative size of the contour length L_{tot} and the bend persistence length A.

9. Consider the discrete one-dimensional model defined by the partition function

$$Z = \sum_{\sigma_1=\pm 1} \cdots \sum_{\sigma_N=\pm 1} \exp\left(\alpha \sum_{i=1}^{N} \sigma_i + \gamma \sum_{i=1}^{N-1} \sigma_i \sigma_{i+1}\right)$$

   Taken as a model for a chain subject to an external stretching force, what is the physical meaning of the parameters α and γ? Show that the average extension of the chain, $\langle z \rangle$, satisfies $\langle z \rangle / L_{\text{tot}} = N^{-1} \mathrm{d} \ln Z / \mathrm{d}\alpha$. Outline how Z, and thereby $\langle z \rangle$, can be calculated by using the transfer matrix method (you may assume that the size is large, $N \to \infty$).

10. The same model as in the previous question is frequently used to describe the helix-coil transition. What is then the meaning of the parameters α and γ? Discuss the temperature dependence of α. How can the helix-coil transition be studied experimentally?

11. Figure 9.10 in Nelson's book shows fractional oxygenation against O_2 concentration for hemoglobin and myoglobin. Explain the difference in behavior between hemoglobin and myoglobin. What is meant by allostery?